

# Space Vector Pulse Width Modulation (SVPWM) Implementation in Induction Motors Through Digitization of Algorithmic Designing

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*Abstract: A number of designs are available for industrial speed control for all kind of motor. Digital design is more preferred in comparison to others as it is most suitable and accurate method of implement. The aim of this paper is to implement the space vector pulse width modulation (SVPWM) principle for the speed control of induction motor and to see the various advantages associated with it. Matlab tool has been used to study the output response of SVPWM based motor control.*

*Keywords: SVPWM, MOSFET, IGBT, Induction Motor, Vector Control.*

## 1. INTRODUCTION

The field oriented vector control is the most compatible and widely used technique for speed control of induction motors. Pulse width modulation (PWM) is used to generate the vector control signal of inverters[1]. Different PWM techniques are available but we have chosen the most recent space vector Pulse width modulation for its stated advantage of low energy consumption and improved transient responses with less generated noises [3]. There are numerous reports about SVPWM implementation on converter using different devices. The focus of studies is on the operating mode and its hardware implementation [1,3,5,7,8,10]. Ying-Yu Tzuo et al. [8] has reported advantages like frequency variation and high switching frequency possibilities on universal ac drives. We have tried to simulate the SVPWM on Matlab for the control of Induction Motor. We have simulated the mathematical equations involved in the scheme. The three phases out of single dc source is also a unique feature of this study. The variable frequency has also found offering better efficiency in comparison with conventional speed control. The algorithm implementation has also been calculated in terms of its bit size and CPU loading. A unique idea has been implemented taking in consideration of all aspects associated with it.

## 2. SPACE VECTOR PULSE WIDTH MODULATION (SVPWM) MATHEMATICAL MODELING

The most popular method to control the speed of AC Motor is known as V/f principle. The magnitude of inverter output at stator terminals of AC motor is kept constant and frequency is variable. This advantage of ~~keeping voltage constant is that it maintains the constant torque.~~

The motor stator vectors are  $V_R$ ,  $V_Y$  and  $V_B$ , the motor is considered at balance load with an unconnected neutral, the voltage across neutral are

$$V_n = (V_R + V_Y + V_B)/3 \quad \dots (1)$$

Voltage across line-neutral

$$V_{Rn} = (V_R - V_n) = (V_{RY} - V_{BR})/3$$

$$V_{Yn} = (V_Y - V_n) = (V_{RY} - V_{BY})/3$$

$$V_{Bn} = (V_B - V_n) = (V_{BY} - V_{BR})/3$$

Voltage across line-line

$$\begin{bmatrix} V_{RY} \\ V_{YB} \\ V_{BR} \end{bmatrix} = (V_{dc}) \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} R \\ Y \\ B \end{bmatrix} \quad \dots(3)$$

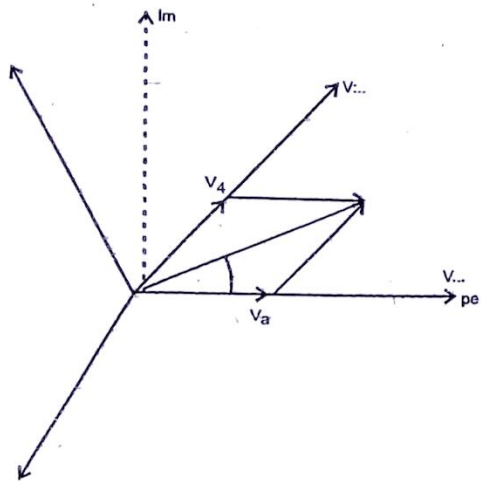


Fig. 1. Vector representation in I Sector

$$\begin{aligned} V_{RY} &= V_{Rdc} - V_{Ydc} \\ V_{YB} &= V_{Ydc} - V_{Bdc} \\ V_{BR} &= V_{Bdc} - V_{Rdc} \end{aligned} \quad \dots (4)$$

From the equation we can determine phase voltage vector  $[V_R, V_Y \text{ and } V_B]$  as follows when  $V_{dc}$  is bus voltage to inverter.

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \frac{1}{3(V_{dc})} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & - \\ - & -1 & 2 \end{bmatrix} \begin{bmatrix} R \\ Y \\ B \end{bmatrix} \quad \dots (5)$$

So maximum phasor voltage

$$V_m = \frac{2}{3} (V_{dc}) \quad \dots (6)$$

Assume one output voltage space vector at a point of time by averaging other five vectors of hexagon with in the same switching period. In rectangular coordinates:

$$T_{SVPWM1} V_m + T_{SVPWM2} V_m \cos \left( \frac{\pi}{3} + j \sin \frac{\pi}{3} \right) \quad \dots (7)$$

$$T_{SVPWM1} \sqrt{\frac{2}{3}} (V_{dc}) \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + T_{SVPWM2} \frac{2}{3} (V_{dc}) \begin{bmatrix} \cos \left( \frac{\pi}{3} \right) \\ \sin \left( \frac{\pi}{3} \right) \end{bmatrix}$$

$$= \frac{T_{SVPWM3}}{2} \sqrt{\frac{2}{3}} (V_{dc}) k \begin{bmatrix} \cos(\pi) \\ \sin(\pi) \end{bmatrix} \quad \dots (8)$$

$$k = |V_m| \sqrt{\frac{2}{3}} V_{dc}, \text{ at } 0^\circ \leq t \leq 60^\circ \quad \dots (9)$$

Now

$$T_{SVPWM1} = \frac{T_{SVPWM}}{2} K \begin{bmatrix} \sin \left( \frac{\pi}{3} \theta \right) \\ \sin \left( \frac{\pi}{3} \right) \end{bmatrix} \quad \dots (10)$$

$$T_{SVPWM2} = \frac{T_{SVPWM}}{2} K \begin{bmatrix} \sin(\theta) \\ \sin \left( \frac{\pi}{3} \right) \end{bmatrix} \quad \dots (10)$$

$$T_{SVPWM1} = T_{SVPWM0} = \frac{T_{SVPWM}}{2} - T_{SVPWM2} - T_{SVPWM1} \quad \dots (12)$$

Reference voltages like  $V_{100}, V_{110}, \dots$  corresponds to one of the switching sector as shown in the Table 1. To obtain the sinusoidal wave voltage output on the motor phases, a continuous rotating vector  $V$  must be produced so that one complete turn of the vector  $v$  on the complex plan represent one electrical turn of the motor (co-incident with one mechanical turn in case on pole pair motor). The circumference described by the rotating vector must stay inside the hexagon as shown in Fig. 4. The goal of space vector is to generate the appropriate PWM signals so that any vector  $V$  inside the hexagon can be produced by time varying the two reference vectors that bound the sector in which  $V$  lies.

$$V = V_a + V_b \quad \dots (13)$$

Where  $V_a$  and  $V_b$  are components of vector  $v$  with respect to  $V_{001}$  and  $V_{011}$  for well defined percentage of PWM period  $T_0$ . Time sequences  $t_a, t_b$  and  $t_0$  are of vector  $V_{001}, V_{011}$  and  $V_{111}$  (or  $V_{000}$ ) inside the PWM period. It can be written as follows :

$$V = V_a + V_b = \frac{t_a}{T_0} V_{001} + \frac{t_b}{T_0} V_{011} + \frac{t_0}{T_0} V_{000} \text{ (or } 111) \quad \dots (14)$$

$$t_a = \frac{V_a}{V_{001}} T_0 \quad \dots (15)$$

$$t_b = \frac{V_b}{V_{011}} T_0 \quad \dots (16)$$

$$t_0 = T_0 - t_a - t_b \quad \dots (17)$$

Table 1. Switching combination showing Line-Line and Line-neutral three phase output voltages

S.No.	R	Y	B	YRN	VYN	VBN	YRY	VYB	VBR	Mag	Ang.
1.	0	0	0	0	0	0	0	0	0	0	0
2.	0	0	1	-1/3	-1/3	2/3	0	-1	1	0.999	-120°
3.	0	1	0	-1/3	2/3	-1/3	-1	1	0	0.999	120°
4.	0	1	1	-2/3	1/3	1/3	-1	0	1	1	180°
5.	1	0	0	2/3	-1/3	-1/3	1	0	-1	1	0
6.	1	0	1	1/3	-2/3	-1/3	1	-1	0	0.999	-60°
7.	1	1	0	1/3	1/3	-2/3	0	1	-1	0.999	60°
8.	1	1	1	0	0	0	0	0	0	0	0

Referring to Fig. 2 it has been converted as follows

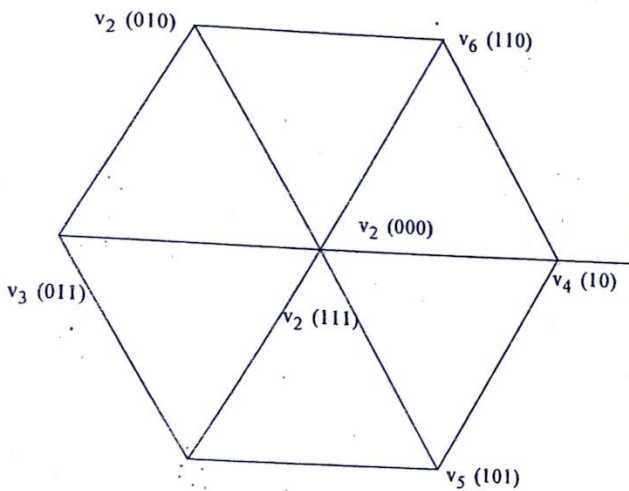


Fig. 2. Hexagon of SVPWM, pattern

$$V \sin = \left( \frac{\pi}{3} - \alpha \right) = V_a \sin \left( \frac{\pi}{3} \right) \quad \dots (18)$$

$$V \sin (\alpha) = V_b \sin \left( \frac{\pi}{3} \right) \quad \dots (19)$$

From which it is possible to get

$$V_a = \frac{2}{\sqrt{3}} V \sin \left( \frac{\pi}{3} - \alpha \right) \quad \dots (20)$$

$$V_b = \frac{2}{\sqrt{3}} V \sin (\alpha) \quad \dots (21)$$

For the three phase two level PWM inverter the switch function is defined as  $SW_i = 1$ , the upper switch

is on and bottom switch is off = 0, the upper switch is off and bottom switch is on.

where  $i = R, Y, B$ .

"1" denotes  $V_{Dc}/2$  at the inverter output, "0" denotes  $-V_{Dc}/2$  at inverter output with respect to neutral point of the d.c. bus. The eight switch states  $Si = (SW_R, SW_Y, SW_B)$  where  $i = 0, 1, \dots, 7$  are shown in Table-1. There are eight voltage vectors  $\bar{V}_0 \dots \bar{V}_7$  corresponding to the switch states  $\bar{S}_0 \dots \bar{S}_7$  respectively. The length lengths of vectors  $\bar{V}_1 \dots \bar{V}_6$  are unity and the length of  $\bar{V}_0$  and  $\bar{V}_7$  are zero. These eight vectors form the voltage vector space as depicted in Fig. 4. The six non-zero voltage space vectors form a hexagonal locus. The vector space is divided into six sectors. It can be seen that when the space vector moves from one corner of the hexagon to another corner, then only the state of one inverter leg has to be changed. The zero space vectors are located at the origin of the reference frame. The reference value of the stator voltage space vector  $\bar{V}_{sref}$  can be located in any of six sectors. Any desired stator voltage space vector inside the hexagon can be obtained from the weighted combination of the eight switching vectors. The goal of the space vector modulation technique is to reproduce the reference stator voltage space vector ( $\bar{V}_{sref}$ ) by using the appropriate switching vectors with minimum harmonic current distortion and the shortest possible cycle time.

$$\bar{V}_{sref} = \frac{T_0}{T_s} \bar{V}_0 + \frac{T_1}{T_s} \bar{V}_1 + \dots + \frac{T_7}{T_s} \bar{V}_7 \quad \dots (22)$$

where  $T_0, T_1, \dots, T_7$  are the turn on time of the vectors  $\bar{V}_0, \bar{V}_1, \dots, \bar{V}_7$  respectively and  $T_0, T_1, \dots, T_7 \geq 0, \sum_{i=0}^7 T_i = T_s$  where  $T_s$  is the sampling time.

In order to reduce the number of switching actions and to make full use of active turn on time for space vectors, the vector  $\overline{V}_{sref}$  is split into the two nearest adjacent voltage vectors and zero vectors  $\overline{V}_0$  and  $\overline{V}_7$  in an arbitrary sector. For Sector 1 in one sampling interval, vector  $\overline{V}_{sref}$  can be given as:

$$\overline{V}_{sref} = \frac{T_1}{T_s} \overline{V}_1 + \frac{T_3}{T_s} \overline{V}_3 + \frac{T_7}{T_s} \overline{V}_7 + \frac{T_0}{T_s} \overline{V}_0 \quad \dots (23)$$

where  $T_s - T_1 - T_3 = T_0 + T_7 \geq 0$ ,  $T_0 \geq 0$  and  $T_7 \geq 0$ . The length and angle of  $\overline{V}_{sref}$  are determined by vectors  $\overline{V}_1, \overline{V}_2, \dots, \overline{V}_6$  that are called active vectors and  $\overline{V}_0, \overline{V}_7$  are called zero vectors. In general

$$\overline{V}_{sref} T_s = T_i \overline{V}_i + T_{i+1} \overline{V}_{i+1} + T_7 \overline{V}_7 + T_0 \overline{V}_0 \quad \dots (24)$$

Where  $T_i, T_{i+1}, T_7, T_0$  are respective on duration of the adjacent switching state vectors ( $\overline{V}_i, \overline{V}_{i+1}, \overline{V}_7$  and  $\overline{V}_0$ ). The on durations are defined as follows:

$$T_i = m T_s \sin(\theta) \quad \dots (25)$$

$$T_{i+1} = m T_s \cos(\theta) \quad \dots (26)$$

$$T_7 + T_0 = T_s - T_i - T_{i+1} \quad \dots (27)$$

Where  $m$  is modulation index defined as:

$$m = \frac{2}{\sqrt{3}} \frac{|\overline{V}_{sref}|}{V_{dc}} \quad \dots (28)$$

$V_{dc}$  is d.c. bus voltage and  $\theta$  is angle between the reference vector  $\overline{V}_{sref}$  and the closest clockwise state vector as depicted in Fig. 4.

In the six step mode, the switching sequence is S1 - S2 - S3 - S4 - S5 - S6 - S1 ..... Further more it should be pointed out that the trajectory of voltage vector  $\overline{V}_{sref}$  should be circular while maintaining sinusoidal input line to line voltage.

In the linear modulation range,  $\overline{V}_{sref} = \sqrt{3}/2 V_{dc}$ , the trajectory of  $\overline{V}_{sref}$  becomes the inscribed circle of the hexagon. In conventional schemes, the magnitude and the phase angle of the reference voltage vector (i.e.  $\overline{V}_{sref}$  and  $\theta$ ) are calculated at each sampling time and then substituted into (27) and (24), (25) to get the value of on duration. Due to Sine Function in (24) and (25) it produces a larger computing delay.

From above modulation index  $m_i = V/V_{xxx}$

$$t_a = \frac{2}{\sqrt{3}} \cdot T_0 \cdot m_i \sin\left(\frac{\pi}{3} - \alpha\right) \dots \dots (29) \quad 0 \leq \alpha \leq \frac{\pi}{3}$$

$$t_b = \frac{2}{\sqrt{3}} \cdot T_0 \cdot m_i \sin(\alpha) \dots \dots (30) \quad 0 \leq \alpha \leq \frac{\pi}{3}$$

When the modulation index exceeds  $\frac{\sqrt{3}}{2}$ , certain values of  $\alpha$  may give negative value of  $t_0$  since this does not have the physical meaning. It can be affirmed that the maximum value of  $m_i$  guarantee the proper working of space vector modulation in linear region is exactly  $\frac{\sqrt{3}}{2}$  higher value of  $m_i$  would lead to over modulation where equation 17, 29 and 30.

Due to some drawbacks of edge alignments it is avoided in this study and center aligned SVPWM pattern is chosen. The value of  $T_0$  is half of the PWM period  $f_s$ .

### III Result and Discussion

From Fig. 3 it is seen that there will be eight possible combination of switching states. Six of them lead to non-zero phase voltages, and two interchangeable states lead to zero phase voltages. When mapped in a 2D-frame fixed to the stator using a Concordia transformation, the six non-zero phase voltages form the vertices of a hexagon. There are eight possible combinations of on and off states for the three upper power transistors. The eight combinations and the derived output line-to-line and phase voltages in terms of DC supply voltage  $V_{dc}$ , according to equations 2 to 4, are shown in Table 1.

$t_a, t_b$  and  $t_0$  is varying continuously inside the space vector hexagon and  $\alpha$  is varying between 0 and  $\frac{\pi}{3}$ . PWM frequency is chosen as 15.625 KHz the registers are 225 pre-scalar to 1 ( $F_{counter} = F_{mtc}/2$ ) it allows acoustic noise elimination generated by switches. The generation is in 8 bit format. Repetition counter has also been fixed at 2 so that a U event is generated every 1.5 PWM periods. The index modulation is adjusted between 0 to 0.866. The output is ranging between 3 to 225 Hz, the  $\alpha$  implies in equation 15 to 17 varies between 0 to  $\frac{\pi}{3}$ .

It is easy to represent any intermediate angle by an 11 bit variable. The most significant 3 bits can be used to indicate the sector, and least significant 8 bits contain the angle of vector  $V$  inside the particular sector the resolution has been chosen by  $\frac{\pi}{3 \cdot 256}$  radians which is equal to 0.234 degrees. But angle  $\alpha$  has been chosen to store in a 16 bit variable so as to keep the resolution frequency high. So finally we fixed the Initial most significant 3 bit to identify the sector the successive 8 bits are used to identify the angle inside the sector and least significant 5 bits are used for the  $t_a, t_b$  and  $t_0$  computation. To reduce the CPU loading a lookup table

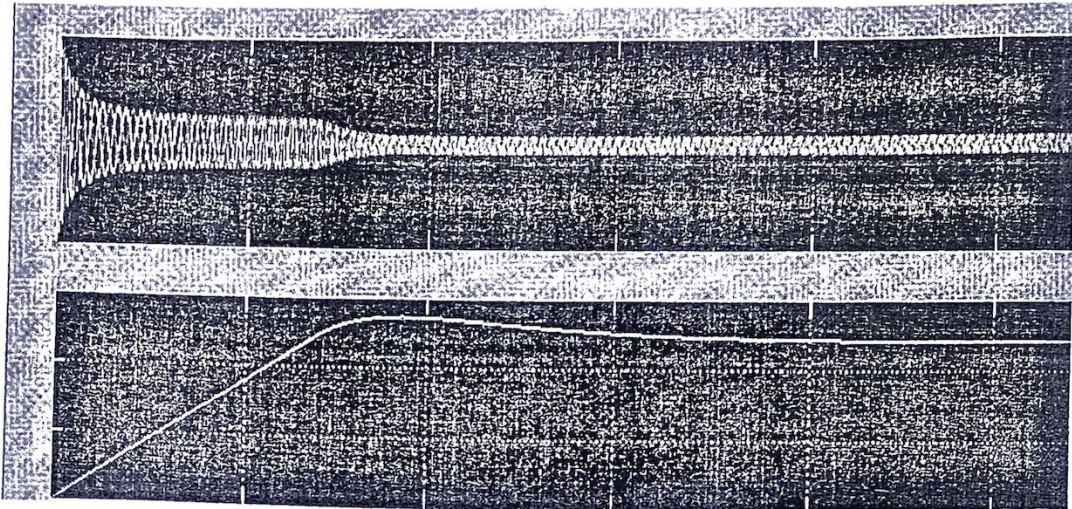


Fig. 5. Simulation Result of terminal current and rotor speed

has been used for storing time  $t_a$ ,  $t_b$  and  $t_0$  with maximum value of modulation index  $m_i$  (0.866).

### Conclusion

It has been observed that the single phase to three phase conversion is accurate and smooth when it is supplied through SVPWM technique. The outputs observed in simulation are nearly sinusoidal wave which shows that energy transfer is there with minimum line losses. The switching noise has also been minimized in this technique with the increased in frequency of PWM, which further helps in reduction in Total Harmonic Distortion (THD). The 16 bit sizing of the angle  $\alpha$  storing with high resolution has considerably reduced the CPU loading. The main advantage observed of this type of implementation is that in digital designing the smooth and accurate fractional changes can be observed.

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