Stochastic Integrated Vendor-Buyer Model with Negative Exponential Crashing Cost

Chandra K. Jaggi
Operational Research,
Faculty of Mathematical Sciences,
New Academic Block, University of Delhi
Delhi
E-mail: ckjaggi@yahoo.com

Neetu Arneja
Department of Computer Science,
Hansraj College, North Campus,
University of Delhi,
Delhi
E-mail: neetuarneja@gamil.com

Haider Ali
Department of Operational Research,
Faculty of Mathematical Science,
New Academic Block, University of Delhi,
Delhi
E-mail: haideralimphil@gamil.com

Abstract: *This study deals with integrated vendor buyer model where vendor's and buyer's perspective are different. Vendor's objective is to reduce materials lead time as far as possible to compete in the market. A vendor produces the ordered quantity and delivered it to the buyer in 'm' number of shipment where buyer's lead time for each order is not the same, which distinguishes this paper from the previous study. Buyer's aim is to earn a good reputation in the market by fulfilling the customer's demand on time with higher service level. The proposed model jointly optimizes buyer's ordered quantity, number of shipments and vendor's lead time. Numerical examples are presented to illustrate the results.*

Index Terms: *Crashing cost, Inventory control, Lead time reduction, Service level*

I. INTRODUCTION

In a highly competitive environment of business, it is observed that mutual coordination between vendor-buyer systems is more profitable as compared to their individual systems. As in the supply [illegible] system, the main objective of the vendor and the [illegible] minimize the joint total expected cost. Goyal [illegible] believed to be the first who introduced the [illegible] the joint optimization. Later, Banerjee [1] inv[illegible] the model with the assumption that the vendor [illegible] ures at a finite rate and considered a lot fo[illegible]. Ha and Kim [7] discussed about the integrate[illegible]-time (JIT) lot-splitting model for smoothing the [illegible]ess by using multiple shipments in small lots. Many researchers (Goyal [20], Goyal and Nebebe [19], Lu [11], Hill [18]) already discussed various models with distinct policies of shipments between the vendor and the buyer. But, the focus was mostly on the production shipment schedule between both parties concerning size and frequency of order under the deterministic scenario i.e. when the lead time and demand are known.

Recently, various researches have proposed integrated inventory models involving variable lead time. Ben-Daya and Hariga [14] examined the integrated single vendor single buyer model with stochastic demand and variable lead time for buyer. Chang et al.[8] extended their model by taking the crashing of ordering cost for the buyer, where lead time and ordering cost are linearly dependent. There are also some integrated inventory models involving variable lead time with quality improvement (Yang and Pan [9], Ouyang et al. [12], Hoque [16], Wu et al. [10], Yu [3]). Unfortunately, none of them considered the reduction of lead time and setup cost for the vendor which helps speed up the production process/delivery of the orders.

Further, Just-In-Time (JIT) philosophy includes the successful execution of all manufacturing activities that are required to produce the product and its fast delivery for an end user. On the other front, it helps in continuous improvement of the manufacturing process and in the elimination of waste, which ultimately help the vendor to provide better product and services at a lesser cost. In recent production management, controllable lead time is important for business success and helps the firm to compete in the market. The JIT philosophy also advocates in favor of comparatively low lead times to order the small lot sizes. Tersine [17] suggested that the materials lead time is composed of different components viz. administration, raw-material requisition, manufacturing, inspection and transportation. Thus, it is completely possible to crash these components at an extra cost. Liao and Shyu [4] presented a probabilistic inventory model in which the order quantity was predetermined and lead time was unique decision variable, which was further extended by Ben-Daya and Raouf [13] with the consideration of the lead time and the ordering quantity as decision variables without considering any shortages. Since then, different authors have presented the stochastic inventory models with lead time reduction (Hahn and Choi [5], Hariga and Ben-Daya [15], Chuang et al.[2]).

In all these articles, the authors concentrated on the benefits driven by reduction of lead time/setup cost, either for the vendor or for the buyer. In this paper, we considered a different situation where a vendor is interested in reducing his materials lead time and setup cost. Buyer's lead time for the first batch is higher as compared to remaining *(m–)* batches. It is assumed that lead time of buyer for batches consists of transportation time only. The demand during lead time is considered to be normal; and the model jointly optimizes the buyer's ordered quantity, lead time and number of shipments and vendor's material lead time and setup cost. Findings are also validated with the help of examples along with the sensitivity analysis.

II. Notations and Assumptions

Notations

D : Average demand per year at the buyer

A_b : Buyer's ordering cost per order

A_v : Vendor's setup cost per setup

L_v : Length of materials lead time

$L(Q)$: Lead time for the buyer

F : Transportation cost of the buyer

b : Fixed delay due to waiting and setup time T_s and transportation time T_b

s : Reorder point of the buyer

h_v : Holding cost per unit per year for the vendor

h_b : Holding cost per unit per year for the buyer

π_J : Vendor's fixed penalty cost per unit short

S : Safety stock

π_J : Buyer's unit shortage cost per unit short

m : The number of shipments in one production cycle, a positive intege

Q : Lot size (order quantity)

P : Production rate at the vendor

Assumptions

1. Inventory is continuously reviewed and buyer places the order whenever the inventory level falls to the reorder s point . The reorder point s = expected demand during lead-time + safety stock, that is $s = DL(Q) + S = DL(Q) + k\sigma\sqrt{L(Q)}$ where is a safety factor.
2. The product is manufactured with a finite production rate P and $P > D$.
3. The lead time for the buyer for first batch $L(q) = (Q/P) + b$.
4. The buyer places an order of size Q and the vendor produces with a finite mQ production rate at one setup, but ship quantity Q to the buyer over times.

III. Mathematical Model

An integrated single vendor – single buyer inventory model for a single commodity has been considered. The vendor's cost and buyer' cost have been given below.

Vendor Cost

The integrated model is planned as follows: the vendor produces mQ units with a finite production rate at one setup and $P > D$, [illegible] quantity Q to the buyer over times. Fig[illegible] the behavior of vendor's inventory p[illegible]'s average inventory is the difference [illegible] accumulated inventory and buyer's acc[illegible]ory as in [8]. Sometimes, it is difficult to break down the lead time into all its components and estimate the duration and cost of each component, thus we have considered aggregate information to solve the purpose of crashing cost estimation. The total crashing cost is negative exponential function of lead time. Thus, the total expected annual cost function for the vendor includes the setup cost, inventory hold[illegible] stock out cost (Penalty or delay during pr[illegible] the lead time crashing cost.

$$TC_v(Q, L_v, m) = \left(\frac{A_v D}{mQ}\right) + \left(\frac{h_v Q}{2}\right)\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \frac{D\pi}{mQ}\sigma_v\sqrt{L_v}\psi(k) + \frac{D}{mQ}\alpha e^{-\beta L_v} \quad ...1$$

Buyer Cost

A continuous review inventory model has been considered where the demand during the stock out period is partially captive. Suppose the buyer places the order at time $t = 0$ when his inventory level reaches the reorder point s, where $s = DL(Q) + k\sigma\sqrt{L(Q)}$. The vendor begins manufacturing the product at time T_s, which includes

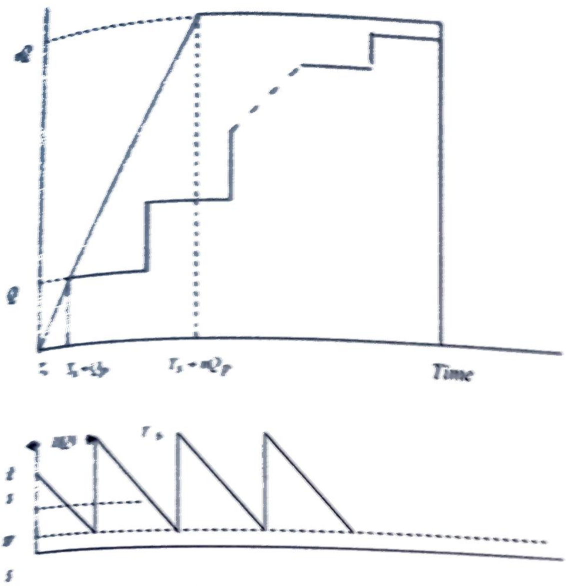

Fig.1 : (a) Vendor's production level and transportation timings (b) Buyer's inventory level and replenishment timings

waiting and setup time, delivers the first batch of products to the buyer at time $T_s + (Q/P)$, and stops the production at time $T_s + (mQ/P)$. The buyer receives the first batch at the time $t_1 = L(Q) = (Q/P) + b$. Since, it is assumed that the production rate is greater than the demand rate, so vendor has sufficient products and able to deliver the second batch at the time $t_2 = t_1 + (Q/D) - T_b$. Therefore, the lead time of the second batch is only T_b. The inventory level of the buyer at time t_2 is $s = DT_b - S$. The time t_3, when [illegible] receive his second batch of [illegible] $= t_2 - T_b = t_2 - (Q/D)$. Hence the lead [illegible] batch is for The total annual expected cost [illegible] is the sum of ordering cost, transportation [illegible] cost and shortage cost.

Therefore, the total annual expected cost for the buyer is given by

$$TC_b(Q,m) = \frac{DA_b}{mQ} + F\frac{D}{Q} + h_b\left(\frac{Q}{2} + S\right) + \frac{\pi_1 D}{mQ}\left[b(s, L(Q)) + (m-1)b(sr, T_b)\right] \quad \ldots 2$$

The expected shortages during the lead time of first batch is given by

$b(s, L(Q)) = \int_s^{\infty}(x_1 - s)f(x_1)dx_1$, x_1 is the demand during the lead time of the first batch with $p.d.f.\ f(x_1)$. We assumed that the lead time demand with is normally distributed with mean $DL(Q)$ and standard deviation $\sigma\sqrt{L(Q)}$, where the safety stock is given by

$$S = k_1\sigma\sqrt{((Q/P)+b)} \quad \ldots 3$$

$$b(s, L(Q)) = \sigma\sqrt{\left(\frac{Q}{P}\right)+b}\ \psi(k_1) \quad \ldots 4$$

where $\psi(k_1) = \int_{k_1}^{\infty}(z - k_1)f(x_1)dx_1$, where k_1 is the safety stock of the first batch.

Similarly, the expected shortages during the lead time of j^{th} batch is given by

$b(sr, L(Q)) = \int_{sr}^{\infty}(x_2 - sr)f(x_2)dx_2$, x_2 is the demand during the lead time of the first batch with $P.D.f.\ f(\chi_2)$ for j - 2,3, ...m. The demand during the lead time of the j^{th} batch is also normally distributed with mean DT_b and the standard deviation $\sigma\sqrt{T_b}$, for The safety stock can also be given as

$$S = k_2\sigma\sqrt{T_b} \quad \ldots 5$$

and expected shortages of the other batches $b(sr, T_b)$ is given by

$$b(sr, T_b) = \sigma\sqrt{T_b}\ \psi(k_2) \quad (6)$$

where k_2 is the safety stock of the j^{th} batch for $j = 2,3,\ldots m$. Obviously $k_1 < k_2$ and $\psi(k_1) > \psi(k_2)$ because $L(Q) > T_b$. Substituting the value of $b(s, L(Q))$ and $b(sr, L(Q))$ from (4) and (6) into (2) provides

$$TC_b(Q,m) = \frac{DA_b}{mQ} + F\frac{D}{Q} + h_b\left(\frac{Q}{2}\right) + h_b k_1\sigma\sqrt{((Q/P)+b)} + \frac{\pi_1\sigma D}{mQ} \times \left[\left(\sigma\sqrt{\left(\frac{Q}{P}\right)+b}\ \psi(k_1) + (m-1)\sqrt{T_b}\ \psi(k_2)\right)\right]$$

Joint Total Expected Cost

Our target is to minimize the joint total expected cost ($JTEC$) per year for the vendor and the buyer i.e.

$Min\ JTEC(Q, L_v, m)$ = Buyer cost + Vendor cost

$Min\ JTEC(Q, L_v, m) =$ [illegible] (7)

In order to find the optimal solution with respect to Q, L_v and m. The necessary conditions are

[illegible]

and [illegible]

[illegible]

Let $S(m) = $ [illegible]

and $T(m) = h_b + $ [illegible]

[illegible] (8)

$\frac{\partial JTEC(Q, L_v, m)}{\partial L_v} = 0 \Rightarrow$ [illegible] (9)

$\frac{\partial JTEC(Q, L_v, m)}{\partial m} = 0 \Rightarrow$ [illegible]

[illegible] (10)

The number [illegible] discrete variable and we [illegible] in order to see the nature [illegible] with respect to it and obtained that [illegible] expected cost is a convex function of m. Moreover [illegible] easily find that joint total expected cost [illegible] is also a convex function of Q and L_v for fixed m. [illegible] obtain the general solutions for the [illegible] variables Q and L_v there is need to set up an iterative procedure by keeping m fixed. Therefore, the following iterative procedure is proposed

Iterative Procedure

Step 1: Set $m = 1$

Step 2: Compute $Q = \left[\sqrt{2DS(m)/T(m)}\right]$, where $[x]$ is the nearest integer to x

Step 3: Substitute the [illegible] based on serv[illegible] in (8) and con[illegible]

Step 4: Set $Q^1 = [Q^1$ [illegible] the value of [illegible] the value of [illegible] $m)$

Step 5: If $|Q^1 - Q| <$ [illegible] otherwise [illegible]

Step 6: Set $m = m$ [illegible] $JTEC(Q_{(m)}, L_v$ [illegible]

Step 7: If $JTEC(Q_{(m)}$ [illegible] $\le JTEC(Q_{(m-1)}$ [illegible] Otherwise, go [illegible]

Step 8: Set (Q, L_v, m) [illegible] (Q, L_v, m) is [illegible]

IV. Numerical Exa[illegible]

In order to illustr[illegible] with the following d[illegible] units/y[illegible] A_b = $50 or[illegible] year, [illegible] $4/unit/ye[illegible] σ_v = 5un[illegible] week, σ = [illegible] = 0.01, [illegible] = $25, α = [illegible]

We consider tha[illegible] with safety factor k = [illegible] as buyers has service [illegible] 95% as a service lev[illegible] which have only lead [illegible] k_1 0.845, $\psi(k_1)$ = 0.11[illegible] and Peterson [6]). Th[illegible] crashing of lead time [illegible]

m	Q	[illegible]
1	1588	[illegible]
2	967	[illegible]
3	704	[illegible]
4	556	[illegible]
5	472	[illegible]

Step 3: Substitute the value of different service factors based on service level set by vendor and buyer in (8) and compute Q^1.

Step 4: Set $Q^1 = \left[Q^1\right]$, if $\left|Q^1 - Q\right| = 0$, then compute the value of lead time L_v using (9) and obtain the value of total expected cost $JTEC\ (Q,\ L_v,\ m)$.

Step 5: If $\left|Q^1 - Q\right| > 0$, then set $Q^I = Q$ and go to step 3, otherwise move to step 6.

Step 6: Set $m = m + 1$, repeat steps 2 - 5 to get $JTEC\,(Q^{'}_{(m)}, L^{'}_{v(m)}, m)$.

Step 7: If $JTEC\,(Q^{'}_{(m)}, L^{'}_{v(m)}, m)$

$\leq JTEC\,(Q^{'}_{(m-1)}, L^{'}_{v(m-1)}, m-1)$, then go to step 6. Otherwise, go to step 8.

Step 8: Set $(Q^{'}, L^{'}_v, m^{'}) = (Q^{'}_{(m-1)}, L^{'}_{v(m-1)}, m-1)$, then

$(Q^{'}, L^{'}_v, m^{'})$ is the optimal solution.

IV. Numerical Example

In ordet to illustrate the model, an inventory system with the foll●wing data has been considered: =1000 units/year, A_b=\$50/order, A_v=\$400/setup, h_b= \$5/unit/year, h_v=\$4/unit/year, π=\$50/unit, π_1=\$100/unit, σ_v=5units/week, σ=7units/week, P=[illegible] units/year, b = 0.01, F = \$25, α = 156, β = 1.

We consider that vendor's s[illegible]evel of 95% with safety factor k = 1.64 and ψ[illegible]2114, where as buyers has service level of 80[illegible]st batch and 95% as a service level for remai[illegible]atches (m-1), which have only lead time of transportation. Therefore, $k_1$0.845, $\psi(k_1)$=0.1!20 and K_2=1.64, =0.02114 (Silver and Peterson [6]). The optimal solutions without the crashing of lead time are presented in Table 1.

Table 1: Optimal solutions without crashing of lead time

time L_v = 8 *weeks*

m	Q	S	$JTEC$	Vendor cost	Buyer cost
1	1740	557	5761	1323	4439
2	1027	333	4140	1483	2657
3	736	242	3505	1565	1940
4	575	192	3165	1615	1550
5	472	159	2954	1648	1306

One can observe from the Table 1 that the reorder level decreases as the number of shipments increases. Then, applying suggested iterative procedure and the crashing of lead time has been done where the lead-time demand follows the normal distribution. The optimal results with crashing of lead time are presented in Table 2. It is found that significant savings could be obtained by the joint effort of both members.

It is observed that the expected cost for both the members reduces with the employment of JIT (Just-in-time) technology whereas buyer generates slightly more benefits as compared to the vendor. It is observed that the ordered quantity and reorder level decreases as the number of shipments increases since buyer would like to place an order frequently instead of keeping large amount of safety stock and moreover he knows that the lead time of batches is comparatively small. It is interesting to find that as the number of shipments is high, then vendor would not like to adopt the crashing of his materials lead time as having enough stock to fulfill the buyer's demand that is why the last row of both the tables Table 1 and Table 2 shows the same optimal solution.

Table 2: Optimal solutions with crashing of lead time

m	Q	s	$JTEC$	L_v	Vendor cost	Buyer cost	Savings (%) Total-cost	Vendor cost	Buyer cost
1	1588	509	5313	5.44	1249	4064	7.78	5.53	8.45
2	967	315	3931	6.19	1420	2511	5.05	4.24	5.49
3	704	232	3377	6.63	1514	1863	3.66	3.31	3.94
4	556	186	3079	6.94	1574	1505	2.71	2.57	2.86
5	472	159	2954	7.77	1648	1306	0.00	0.00	0.00

Table 3 represents the results obtained when the service level of vendor becomes 98% instead of 95%. Similarly, Table 4 shows the results obtained when the level of service of buyer becomes 95% for first batch and 95% for rest of the batches.

Findings from Table 3 and Table 4 clearly show that the vendor's and buyer's cost increases with the increase in their service levels.

Table 3: Optimal results when vendor's service level =98%

m	Q	s	$JTEC$	L_v	Vendor cost	Buyer cost
1	2733	868	8766	4.00	1864	6902
2	1910	610	7336	4.78	2499	4836
3	1366	440	6145	5.23	2666	3479
4	1065	345	5496	5.55	2763	2732
5	799	262	4684	5.79	2604	2079

Table 4: Optimal results when buyer's service level =95% for first batch and 95% for other batches

m	Q	S	$JTEC$	L_v	Vendor cost	Buyer cost
1	1736	560	5749	5.44	1320	4429
2	1025	336	4136	6.19	1481	2655
3	734	244	3500	6.63	1562	1938
4	574	194	3164	6.94	1613	1550
5	472	162	2957	7.18	1648	1309

V. Conclusion

This paper presents integrated vendor-buyer model that analyzes the returns obtained by employing Just-In-Time (JIT) technology when the crashing cost of lead time is negative exponential and carried out on vendor's part. Further, it is assumed that buyer's lead time for the first batch is higher as compared to remaining batches as the production rate is generally higher than the demand rate at the vendor side. The joint total expected cost of both the parties includes the setup cost, ordering cost, holding cost, stock out cost and lead-time crashing cost. The overall cost of the integrated inventory system reduces with the cooperation of both parties. Moreover, findings clearly show that significant savings could be obtained by crashing the components of the lead time for the vendor.

References

(1) A. Banerjee, "A joint economic lot size model for purchaser and vendor", Decision Sciences, vol 17, 1986, pp. 292–311.

(2) B.R. Chuang, L.Y. Ouyang and Y.J. Lin, "A minimax distribution free procedure for mixed inventory model with backorder discounts and variable lead time", *Journal of Statistics and Management Systems*, vol. 7, No.1, 2004, pp. 65–76.

(3) C.H. Yu, "A note on integrated single vendor single buyer model with stochastic demand and variable lead time", *International Journal of Production Economics*, vol. 114, 2008, pp. 294–297.

(4) C.J. Liao and C.H. Shyu, "An analytical determination of lead time with normal demand", International *Journal of Operations & Production Management*, vol.11, 1991 pp.72–78.

(5) C.K. Hahn and T.Y. Choi, "Positive management strategy for materials lead time", in *Proc. 51st Annu. International Conf.* Chicago-IL, 1996

(6) E.A. Silver and R. Peterson, *Decision Systems for Inventory Management and Production Planning*, New York: John Wiley, 1985

(7) H.A. [illegible] ementation of JIT [illegible] *[illegible] ction Planning* [illegible] –157.

(8) H.C. Chang [illegible] C.H. Ho, "Integrated vendor–buyer [illegible] inventory models with controllable lead time and [illegible]", *European Journal of Operational Research* [illegible] No. 2, 2006, pp. 481–495.

(9) J.S. Yang [illegible] purchasing: an integrated inventory [illegible] variable lead time and [illegible] *International Journal of Produ*[illegible] 5, 2004, pp. 853–863.

(10) J.W. Wu [illegible] "Computational algorithmic procedure of optimal [illegible] policy involving a negative exponential crashing cost and variable lead time demand", *Applied Mathematics and Computation*, vol. 184, 2007, pp. 798–808.

(11) L. Lu, "A one-vendor multi-buyer integrated inventory model", *European Journal of Operational Research*, vol. 81, 1995, pp. 312–323.

(12) L.Y. Ouyang, K.S. Wu and C.H. Ho, "An integrated vendor–buyer inventory model with quality improvement and lead time reduction", *International Journal of Production Economics*, vol.108, 2007, pp. 349–358.

(13) M. Ben-Daya and A. Raouf, "Inventory models involving lead time as decision variable", *Journal of the Operational Research Society*, vol. 45, No. 5, 1999, pp. 579–82.

(14) M. Ben-Daya and M. Hariga, "Integrated single vendor single buyer model with stochastic demand and variable lead time", *International Journal of Production Economics*, vol. 92, 2004, pp. 75–80.

(15) M. Hariga and M. Ben-daya, "Some stochastic inventory models with deterministic variable lead time", *European Journal of Operational Research*, vol.113, 1999, pp. 42–51.

(16) M.A. Hoque, "An alternative model for integrated vendor-buyer inventory under controllable lead time and its heuristic solution", *International Journal of Systems Science*, vol. 38, No. 6, 2007, pp. 501–509.

(17) R.J. Tersine, *Principles of Inventory and Materials Management*. New Jersey, PTR Prentice Hall, Englewood Cliffs, 1994.

(18) R.M. Hill, "The optimal production and shipment policy for the single-vendor single-buyer integrated production inventory problem", *International Journal of Production Research*, vol. 37, No.11, 1999, pp. 2463–2475.

(19) S.K. Goyal and F. Nebebe, "Determination of economic production- shipment policy for a single-vendor-single-buyer system", *European Journal of Operational Research*, vol. 121, 2000, pp. 175–178.

(20) S.K. Goyal, "A one-vendor multi-buyer integrated inventory model: a comment", *European Journal of Operational Research*, vol. 82, 1995 pp. 209–210.

(21) S.K. Goyal, "An integrated inventory model for a single supplier–single customer problem", *International Journal of Production Research*, vol. 15, No. 1, 1976, pp.107–111.

(22) Y.J. Lin, "An integrated vendor-buyer inventory model with backorder price discount and effective investment to reduce ordering cost", *Computers and Industrial Engineering*, vol. 56, 2009, pp. 1597–1606.

❐