Granular Computing Approach for Handling Uncertainty in Optimization Problems

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Abstract: *Optimization problems got a lot of attention from many researchers; in real world application, there is always uncertainty in problem specification, interval numbers, fuzzy numbers, and rough numbers play important roles in representing uncertain quantities but these heterogeneous types of numbers are forming a challenge in computation. This paper proposes a Unified Granular Number (UGN) that we call G- Number to act as a general form for any uncertain number. G- Number represents higher level of abstract that hold only common properties of different types of uncertain granular numbers while ignoring some particular properties which are not necessary to be considered in such higher abstract level. This paper shows a solution for Uncertain Traveling Salesman Problem (UTSP) also shows a modification for Dijkstra's algorithm to manipulate different uncertain numbers by applying the idea of G- number; the main benefit of using such a proposed G- number is the ability to represent all types of uncertain numbers using unified formality that greatly simplifies arithmetic operations. The results are compared to the solutions in crisp cases.*

Keywords*: Optimization, Granularity, Granular Computing, Unified Granular Number, Fuzzy Set, Interval, Rough Set, Uncertainty, TSP, Dijkstra's Algorithm.*

I. INTRODUCTION

The information that people obtain is usually uncertain and incomplete. In recent years, the definition of uncertainty has been emerged from different perspectives. Young R.C. (1931) and Moore R E. (1979) proposed the theory of interval set [1], [2]. Zadeh (1965) proposed fuzzy set theory [3], then in 1975 he introduced type-2 fuzzy set and even higher types [4]. Pawlak (1982) proposed the rough set theory [5]. All the previously sets theories are constructs falling under the same umbrella of Granular Computing (GrC) for handling various types of uncertainty. The theory of granular computing has recently emerged as a coherent conceptual and algorithmic platform aimed at the representation and processing of information granules [6].

Granular computing may be regarded as a label of theories, methodologies, techniques, and tools that make use of granules, i.e., groups, classes, or clusters of a universe, in the process of problem solving [7]. Generally speaking, information granules are collections of entities that usually originate at the numeric level and are arranged together due to their similarity, functional or physical adjacency, indistinguishability, coherency, or the like [8]. At present, granular computing is more a theoretical perspective than a coherent set of methods or principles [9].

Classical approaches for information processing use exact and precise algorithms that manipulate only confident data and precise numbers. Intervals, fuzzy numbers and rough numbers as types of uncertain granular numbers are very useful forms in representing uncertain quantities, to solve problems that containing uncertain data or inexact numbers, however, this gives us more responsibilities of developing new solutions to problems which contain such types of uncertain numbers in their specifications.

II. UNIFIED GRANULAR NUMBER

G- Number is an extension of a regular number in the sense that it does not refer to one single value but rather it refers to a connected set of possible values. G-Number is a quantity whose value is imprecise, rather than exact as the case with "ordinary" (single-valued) numbers. G-number X is expressed by three factors which are: the center value of number x, radius of number r_x and $A(X)$ which represents the covered area of number X as in form $X = G\frac{(x, r_x)}{A(X)}$. This formulation of G-number represents the most common properties of uncertain granular number which are lower and upper boundaries, furthermore the weight of number is also considered to distinguish among different numbers within same boundaries [10].

III. TRANSFORMATION TO G-NUMBER

This subsection shows the process of transforming various types of uncertain numbers such as interval number, fuzzy number and rough number and also crisp number to the proposed form of G-Number.

A. Interval

Any interval number X = [*a*, *b*] could be transformed into G- Number form, since x = 1/2 (a+b) is a center value of number *X*, r_X = 1/2 (b - a) is a radius of number *X* and A (X) = (b - a): represents the weight of number X which is its area see figure 1.

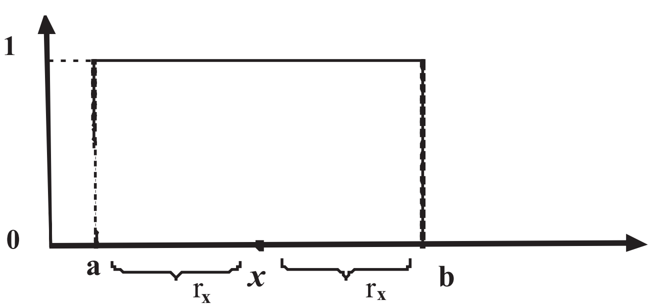

Fig. 1: Interval number A = [a, b]

The new formula for equivalent unified granular number is:

$$X = G\frac{(\frac{1}{2}(a + b), \frac{1}{2}(b - a))}{b - a}$$

B. Fuzzy Number

Any type of fuzzy number *X* could be transformed into G- Number form, since *X, x* is a center value of number *X*, is a radius of number *X and A (X)* : represents the weight of number X which is its area.

The following cases consequently arise:

a) For triangular fuzzy number *X*, as shown in Fig. 2, which, it can be denoted by *X = Tri (a, b, c)* with membership function

$$\mu_X(x) = \max\left(\min\left(\frac{x - a}{b - a}, \frac{c - x}{c - b}\right), 0\right)$$

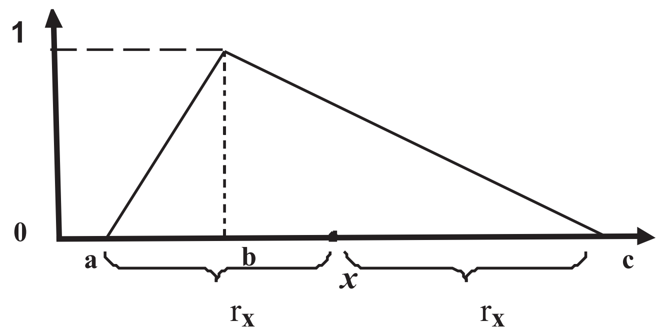

Fig. 2 : Triangular fuzzy number X

The new formula for equivalent unified granular number is:

$$X = G\frac{((a + c) / 2, (c - a) / 2)}{(c - a) / 2}$$

b) For trapezoidal fuzzy number Y, as shown in Fig. 3, which can be denoted by: *Y = Trap (a, b, c, d)*, with membership function:

$$\mu_Y(x) = \max\left(\min\left(\frac{x - a}{b - a}, 1, \frac{c - x}{c - b}\right), 0\right)$$

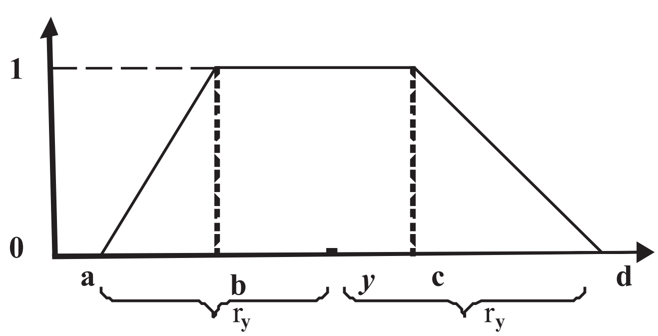

Fig. 3 : Trapezoidal fuzzy number Y

The new formula for equivalent unified granular number is:

$$Y = G \frac{((a + d) / 2, (d - a) / 2)}{^1/_2 ((d - a) + (c - b))}$$

C. Rough Number

Any type of rough number $X = (\underline{X}, \overline{X}) = ([c, d], [a, b])$ could be transformed into G- number form such that: x is a center value of upper approximate $\overline{X}$, is a radius of upper approximate $\overline{X}$ and A (X) represents the weight of number X, in case of rough number there are two considered lengths which are lower approximations length L ($\underline{X}$) and upper approximations length L ($\overline{X}$), to calculate the weight of number we suggest the average of these two lengths

$A(X) = \frac{1}{2}$ (L (X) + L (X)), see figure 4.

The new formula for equivalent unified granular number is:

$$X = G \frac{(^1/_2 (a + b), ^1/_2 (b - a))}{^1/_2 ((b - a) + (c - d))}$$

D. Crisp Number

For any crisp number X , it is trivial to recognize the equivalent unified granular number formula, which is: $X = G \frac{(X,\ 0)}{0}$.

Same formula could be deduced from the equivalent unified granular number formula for interval number by setting the upper limit ‘a’ and the lower limit ‘b ’for interval by the same number X. Also this formula could be deduced from the equivalent unified granular number formula for any type of fuzzy number by setting its support by zero, same formula could be deduced from the equivalent unified granular number formula for any type of any rough number by setting upper approximation $\overline{X}$ and lower approximation $\underline{X}$ by the same exact number.

IV. ADDITION OPERATION ON G - NUMBERS

For any two G- numbers $X = G \frac{(X,\ r_x)}{A(X)}$ and $Y = G \frac{(y,\ r_y)}{A(Y}$ the addition operation is defined as follows:

$$X + Y = G \frac{(x + y, r_x + r_y)}{A(X) + A(Y)}$$

To proof this formula, we check it for each type of uncertain granular number:

A. In case of crisp numbers

$r_x = A(Y) = 0$, $r_y = A(Y) = 0$ so $X = G \frac{(x,\ 0)}{0} = x$ and $Y = G \frac{(y,\ 0)}{0} = y$

$$X + Y = G \frac{(x,\ 0)}{0} + G \frac{(y,\ 0)}{0} = x + y$$

$$G \frac{(x + y, r_x + r_y)}{A(X) + A(Y)} = G \frac{(x + y, 0)}{0} = x + y$$

B. In case of interval numbers

Suppose $X = [a, b] = G \frac{(x,\ r_x)}{A(X)}$ such as $x = \frac{a + b}{2}$, $r_x = \frac{b - a}{2}$ and $A(X) = b - a$

Suppose $Y = [c, d] = G \frac{(y,\ r_y)}{A(Y)}$ such as $y = \frac{c + d}{2}$, $r_y = \frac{d - c}{2}$ and $A(Y) = d - c$

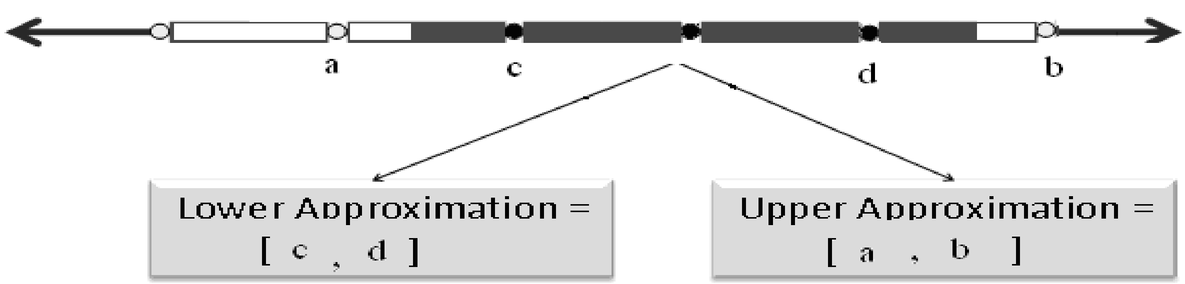

Fig. 4: Rough number *X* = ([*c*, *d*], [*a*, *b*])

$X + Y = [a, b] + [c, d] = [a + c, b + d]$

$\text{Center}(X + Y) = \frac{1}{2}(a + b + c + d) = \frac{1}{2}(a + b) + \frac{1}{2}(c + d) = x + y.$

$r_{x+y} = \frac{1}{2}((b + d) - (a + c)) = \frac{1}{2}((b - a) + (d - c)) = r_x + r_y$

$A(X + Y) = (b + d) - (a + c) = (b - a) + (d - c) = A(X) + A(Y)$

C. In case of triangular fuzzy numbers

Suppose $X = G\,\frac{(x,\ r_x)}{A(X)} = \text{Tri}(x - r_x, b_1, x + r_x)$ and $A(X) = r_x$ see Fig. 5.

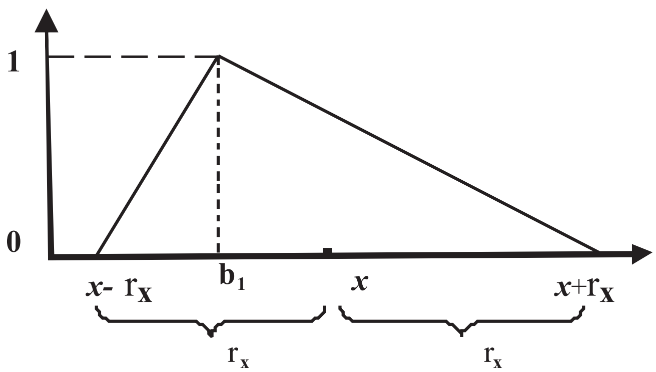

Fig. 5: $X = \text{Tri}(x - r_x, b_1, x + r_x)$

Suppose $Y = G\,\frac{(y,\ r_y)}{A(Y)} = \text{Tri}(y - r_y, b_2, y + r_y)$ and $A(Y) = r_y$ see Fig. 6.

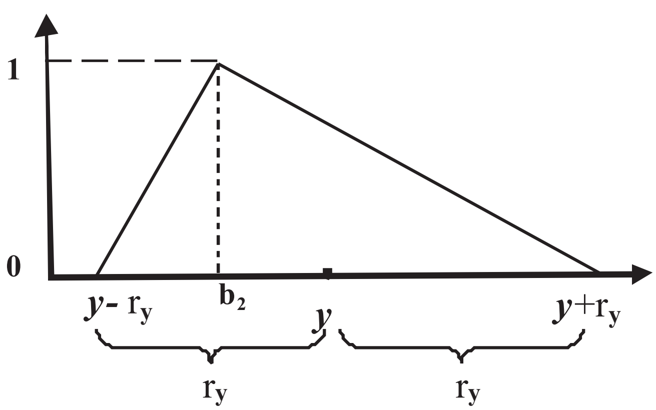

Fig. 6: $Y = \text{Tri}(y - r_y, b_2, y + r_y)$

$X + Y = G\,\frac{(x,\ r_x)}{A(X)} + G\,\frac{(y,\ r_y)}{A(Y)} = \text{Tri}(x - r_x, b_1, x + r_x) + \text{Tri}(y - r_y, b_2, y + r_y)$

$= \text{Tri}((x - r_x) + (y - r_y), b_1 + b_2, (x + r_x) + (y + r_y))$

$= \text{Tri}((x + y) - (r_x + r_y), b_1 + b_2, (x + y) + (r_x + r_y))$

$= G\,\frac{(x + y,\ r_x + r_y)}{A(X + Y))}$

$A(X + Y) = r_x + r_y = A(X) + A(Y)$ as shown in Fig. 7.

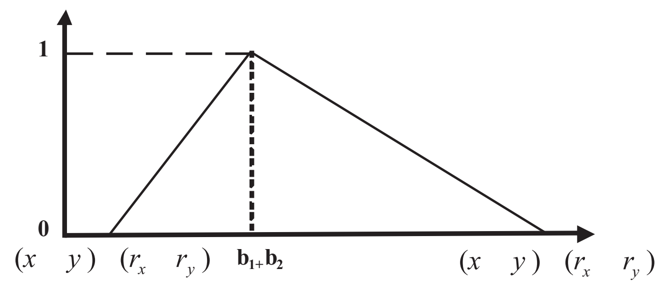

Fig. 7: X + Y (triangular fuzzy numbers)

D. In case of trapezoidal fuzzy numbers

Suppose $X = G\,\frac{(r,\ r_x)}{A(X)} = \text{Trap}(x - r_x, b_1, c_1, x + r_x)$ and $A(X) = r_x + \frac{1}{2}(c_1 - b_1)$ see Fig. 8.

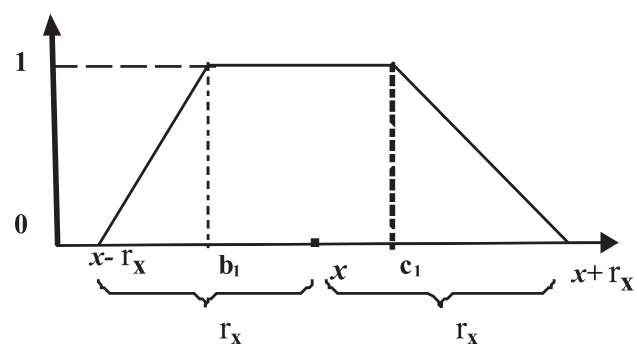

Fig. 8: $X = \text{Trap}(x - r_x, b_1, c_1, x + r_x)$

Suppose $Y = G\,\frac{(y,\ r_y)}{A(Y)} = \text{Trap}(y - r_y, b_2, c_2, y + r_y)$ and $A(Y) = r_y + \frac{1}{2}(c_2 - b_2)$ see Fig. 9.

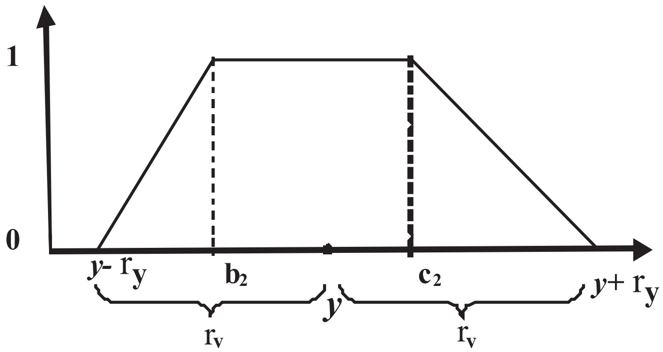

Fig. 9: Y = Trap ($y - r_y$, b_2, c_2, $y + r_y$)

$X + Y = G\ \dfrac{(x,\ r_x)}{A\,(X)} + G\ \dfrac{(y,\ r_y)}{A\,(Y)}$ = Trap ($x - r_x$, b_1, c_1, $x + r_x$) + Trap ($y - r_y$, b_2, c_2, $y + r_y$)

= Trap (($x - r_x$) + ($y - r_y$), $b_1 + b_2$, c_1, c_2 ($x + r_x$) + ($y + r_y$))

= Trap (($x + y$) - ($r_x + r_y$), $b_1 + b_2$, c_1, c_2, ($x + y$) + ($r_x + r_y$))

$= G\ \dfrac{(x + y,\ r_x + r_y)}{A\,(X + Y))}$

$A\,(X + Y) = r_x + r_y + 1/2\ ((c_1 + c_2) - (b_1 + b_2))$

$= rx + 1/2\ (c_1 - b_1) + r_y + 1/2\ (c_2 - b_2)$

= A (X) + A (Y) as shown in figure 10.

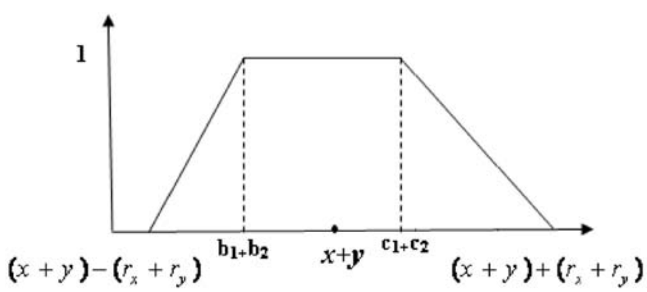

Fig. 10: X+Y (trapezoidal fuzzy numbers)

Some types of fuzzy numbers such as Gaussian fuzzy number, it is suitable to prove the formula of addition by using alpha-cut method or approximate each of them to trapezoidal fuzzy number as a general case.

E. In case of rough number

Suppose that $X = (\underline{X}, \overline{X}) = ([a_1, b_1,], [c_1, d_1] = G\ \dfrac{(x,\ r_x)}{A\,(X)}$ such that: $x = {}^1/_2\ (d_1, + c_1)$, $r_x = {}^1/_2\ (d_1 - c_1)$ and $A\,(X = {}^1/_2\ (L\,(\underline{X}) + 1\ L\,(\overline{X}))$ since $L\,(\underline{X}) = b_1 - a_1$, $L\,(X) = d_1 - c1$

Suppose that

$Y = (Y, \overline{Y}) = ([a_2, b_2], [c_2, d_2]) = G\ \dfrac{(y,\ r_y)}{A\,(Y)}$

such that: $y = {}^1/_2\ (d_2 + c_2)$, $r, = {}^1/_2\ (d_2 - c_2)$

and $A(\overline{Y}) = {}^1/_2\ (L\,(\underline{Y}) + L\,(\overline{Y}))$ since $L\,(\underline{Y}) = b_2 - a_2$, $L\,(\overline{Y}) = d_2 - c_2$

Therefore $X + Y = (\underline{X} + \underline{Y}, \overline{X} + \overline{Y}) = ([a_1 + a_2, b_1 + b_2], [c_1 + c_2, d_1 + d_2])$

Center $(X + Y) = {}^1/_2\ (d_1 + d_2 + c_1 + c_2) = {}^1/_2\ (d_1 + c_1) + {}^1/_2\ (d_2 + c_2) = x + y$

$r_{x+y} = {}^1/_2\ ((d_1 + d_2) - (c_1 + c_2)) = {}^1/_2\ (d_1 - c_1) + {}^1/_2\ (d_2 - c_2) = r_x + r_y$

$L\,(\underline{X + Y}) = (b_1 + b_2) - (a_1 + a_2) = (b_1 - a_1) + (b_2 - a_2) = L\,(X) + L\,(Y)$

$L\,(\overline{X + Y}) = (d_1 + d_2) - (c_1 + c_2) = (d_1 - c_1) + (d_2 - c_2) = L\,(X) + L\,(\overline{Y})$

$A\,(X + Y) = {}^1/_2\ (L\,(\underline{X + Y}) + L\,(\overline{X + Y})) = {}^1/_2\ (L\,(\underline{X}) + L\,(\underline{Y}) + L\,(\overline{X}) + L\,(\overline{Y}))$

$= {}^1/_2\ (L\,(\underline{X}) + L\,(\overline{X})) + {}^1/_2\ (L\,(\underline{Y}) + L\,(\overline{Y}))$

= A (X) + A (Y).

V. G – NUMBER BASED SOLUTIONS FOR OPTIMIZATION PROBLEMS

Traveling Salesman problem (TSP) and Shortest Path Problem are categorized as optimization problems which have been studied in operations research and theoretical computer science [11]. There many algorithms are introduced to solve such a kind of problems such as Dijkstra's algorithm [12], Bellman–Ford algorithm [13], Floyd–Warshall algorithm [14] and Johnson's algorithm [15], ant colony optimization algorithm [16] and honey- bee mating optimization [17], some solutions for these two problems are introduced under fuzziness uncertainty [18], in next subsections we introduce solutions for these two problems under various types of uncertainty

A. Uncertain Travelling Salesman Problem (UTSP)

The salesman has to visit all cities and return to home at the end of journey, given a list of cities and their pairwise distances, which are different types of uncertain numbers.

Example: This example shows how G-Numbers could replace different types of uncertain numbers in traveling salesman problem. The set of cities is C = $\{C_1, C_2, C_3, C_4\}$ as shown in figure 11.

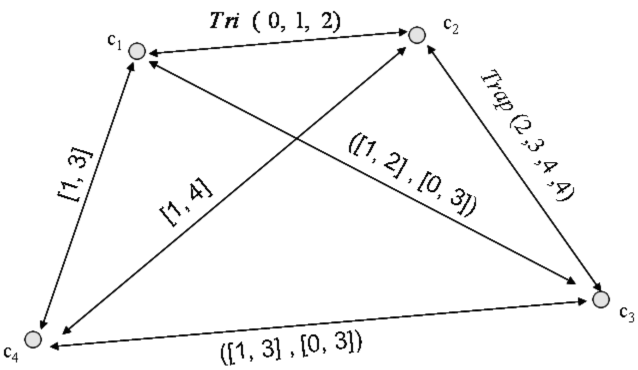

Fig. 11: Distance between cities as various uncertain numbers (UTSP)

D: is the distance function, it can be defined as following:

$D(C_1, C_2)$ = Tri (0, 1, 2)

$D(C_2, C_3)$ = Trap (2, 3, 4, 4)

$D(C_3, C_4)$ = ([1, 3], [0, 3])

$D(C_1, C_4)$ = [1, 3)

$D(C_1, C_3)$ = ([1, 2], [0, 3])

$D(C_2, C_4)$ = [1, 4]

To find the shortest cycle, we-will use the representation of unified granular number as follows:

Tri (0, 1, 2) = $G\,\frac{(1,1)}{1}$

Trap (2, 3, 4, 4)] = $G\,\frac{(3,\ 1)}{1.5}$

([1, 3], [0, 3]) = $G\,\frac{(1.5,\ 1.5)}{2.5}$

[1, 3] = $G\,\frac{(2,1)}{1}$

([1, 2], [0,3]) = $G\,\frac{(1.5,\ 1.5)}{2}$

[1, 4] = $G\,\frac{(2.5,\ 1.5)}{3}$

By calculating the lengths of all possible paths then ranking them according to the value of center point, we can say the shortest cycle is:

C1 → C2 → C4 → C3 → C1

The lenght of this cycle can be calculated as follows:

$$G\,\frac{(1,\ 1)}{1} + G\,\frac{(2.5,\ 1.5)}{3} + G\,\frac{(1.5,\ 1.5)}{2.5} + G\,\frac{(1.5,\ 1.5)}{2} = G\,\frac{(6.5,\ 5.5)}{8.5}$$

The single value which reflects previous total distance is : 6.5, which is the same result in case of crisp number.

B. G-Number Based Dijkstra's Algorithm

Dijkstra's algorithm is designed to determine the shortest routes between the source node and every other node in the network [12]. In our case the distances between the nodes are represented by different types of uncertain numbers such as: interval numbers, fuzzy numbers, rough numbers and also some of them could be represented by classical real numbers. These heterogeneous types of numbers are forming a challenge in calculation the shortest path. To overcome this challenge we convert all types of numbers into G-numbers, and then they could be calculated and ranked according to their centers values.

G-Number Dijkstra's algorithm: Let u_i be the shortest distance from source node 1 to node $\boldsymbol{i}$, and define $D_{ij} = G\,\frac{(x,\ r_x)}{A(X)}$, $x \geq 0$ the length of arc (i, j).

The algorithm defines the label for an immediately succeeding node j as $[u_j, \boldsymbol{i}] = [u_i + D_{ij}, \boldsymbol{i}]$. The label for the starting node is [0, ⁻], indicating that the node has no predecessor. Node labels in Dijkstra's algorithm are of two types: temporary and permanent. A temporary label is modified if a shorter route to a node can be found. If no better route can be found, the status of the temporary label is changed to permanent.

- **Step 0:**

a) Transform all distances from original forms to G-number form.

b) Label the source node (node 1) with the permanent label $[G \frac{(0,0)}{0}, -]$.

c) Set $i = 1$.

- **Step *1:***

(a) Compute the temporary labels $[u_i + D_{ij}, i]$ for each node j that can be reached from node i, provided j is not permanently labeled. If node j is already labeled with $[[u_i + D_{ij}, i], k]$ through another node k and, if center (ui + D_{ij}) < *center* (u_j) , replace $[u_j, k]$ with $[u_i + D_{ij}, i]$.

(b) If all the nodes have permanent labels, stop. Otherwise, select the label $[u_r, s]$ having the shortest distance (= u_r) among all the temporary labels (break ties arbitrarily). Set $i = r$ and repeat step i.

Example: The network in figure 12 gives the permissible routes and their lengths between city 1 (node 1) and four other cities (nodes 2 to 5) as follows:

D_{12} = Tri (80, 100, 110)

D_{13} = [25, 35]

D_{23} = Trap (10, 15, 25, 30)

D_{34} = (10, [6, 16])

D_{35} = 60

D_{42} = [10, 18]

D_{45} = ([35, 55], [30, 66])

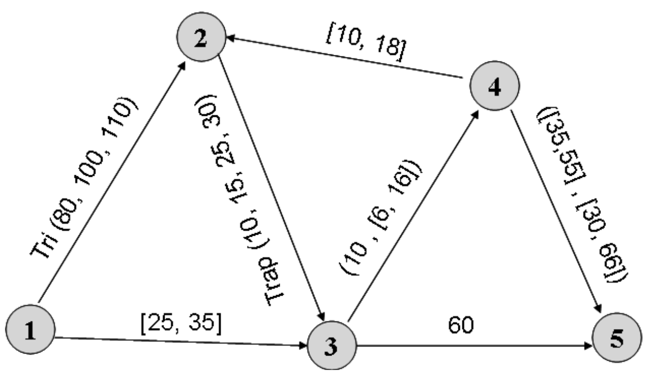

Fig. 12: The permissible routes and their lengths between nodes

There are two distances are represented as fuzzy numbers, triangular fuzzy number in the distance from node 1 to node 2, and trapezoidal fuzzy number in the distance form node 2 to node 3, also there are two distance are represented as interval numbers in the distance from node 1 to node 3 and in the distance form node 4 to node 2, there are two distance are represented as rough numbers in the distance from node 3 to node 4 and in the distance form node 4 to node 5, and there is a distance is represented as crisp number from node 3 to node 5. We need to determine the shortest routes between city 1 and each of the remaining four cities.

Iteration 0:

a) Transform all distances from original forms to G-number form as follows:

Tri (80, 100, 110) = $G \frac{(95, 15)}{15}$

[25, 35] = $G \frac{(30, 5)}{10}$

Trap (10, 15, 25, 30) = $G \frac{(20, 10)}{15}$

(10, [6, 16]) = $G \frac{(11, 5)}{5}$

60 = $G \frac{(60, 0)}{0}$

[10, 18] = $G \frac{(14, 4)}{8}$

([35, 55], [30, 66]) = $\frac{(48, 18)}{28}$

b) Assign the permanent lable $[G \frac{(0,0)}{0}, -]$ to node 1 see figure 13.

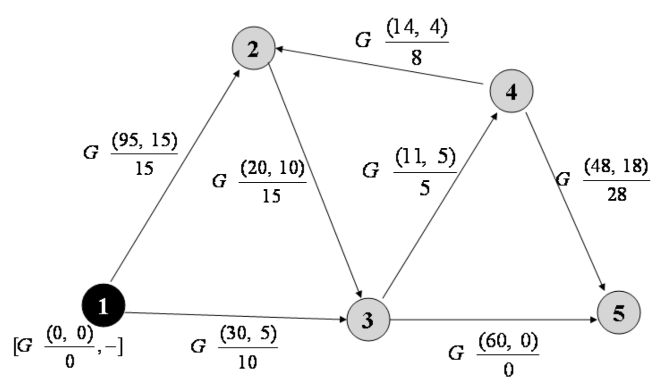

Fig. 13: Iteration 0

Iteration 1:

Nodes 2 and 3 can be reached from (the last permanently labeled) node 1. Thus, the list of labeled nodes (temporary and permanent) becomes as presented in Table 1.

Table 1: Iteration 1

Node	Lable	Status
1.	$\left[G\ \frac{(0,0)}{0}, -\right]$	Permanent
2.	$\left[G\ \frac{(0,0)}{0} + G\ \frac{(95,\ 15)}{15}, 1\right] = \left[G\ \frac{(95,\ 15)}{15}, 1\right]$	Temporary
3.	$\left[G\ \frac{(0,0)}{0} + G\ \frac{(30,\ 5)}{10}, 1\right] = \left[G\ \frac{(30,\ 5)}{10}, 1\right]$	Temporary

For the two temporary labels $[G\ \frac{(95,\ 15)}{15}\ 1]$ and $[G,\ \frac{(30,\ 5)}{10}\ 1]$, node 3 yields the smaller distance ($u_3 = \frac{(30,\ 5)}{10}G$) see Figure 14. Thus, the status of node 3 is changed to permanent.

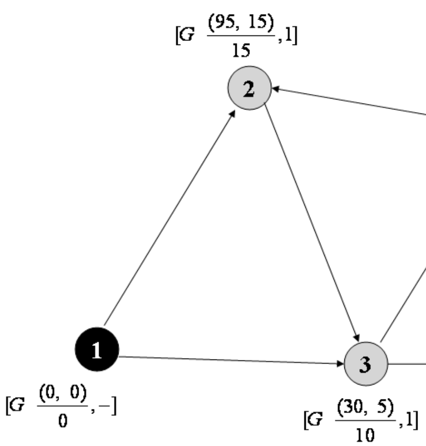

Fig. 14: Iteration 1

Iteration 2:

Nodes 4 and 5 can be reached from node 3 see Figure 15, and the list of labeled nodes becomes as presented in Table 2.

Table 2: Iteration 3

Node	Lable	Status
1.	$\left[G\ \frac{(0,0)}{0}, 1\right]$	Permanent
2.	$\left[G\ \frac{(95,\ 15)}{15}, 1\right]$	Temporary
3.	$\left[G\ \frac{(30,\ 5)}{10}, 1\right]$	Permanent
4.	$\left[G\ \frac{(30,5)}{10} + G\ \frac{(11,\ 5)}{5}, 3\right] = \left[G\ \frac{(41,\ 10)}{15}, 3\right]$	Temporary
5.	$\left[G\ \frac{(30,0)}{10} + G\ \frac{(60,\ 0)}{0}, 3\right] = \left[G\ \frac{(90,\ 5)}{10}, 3\right]$	Temporary

The status of the temporary label $[G\ \frac{(41,\ 10)}{15},\ 3]$ at node 4 is changed to permanent ($u_4 = G\ \frac{(41,\ 10)}{15}$).

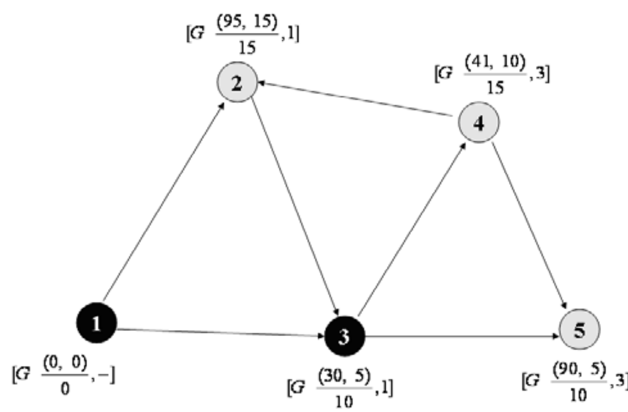

Fig. 15. Iteration 2

Iteration 3:

Nodes 2 and 5 can be reached from node 4. Thus, the list of labeled nodes is updated as shown in Table 3.

Table 3: Iteration 3

Node	Lable	Status
1.	$\left[G\ \frac{(0,0)}{0}, -\right]$	Permanent
2.	$\left[G\ \frac{(41,10)}{15} + G\ \frac{(14,\ 4)}{8}, 4\right] = \left[G\ \frac{(55,\ 14)}{23}, 4\right]$	Temporary
3.	$\left[G\ \frac{(30,\ 5)}{10}, 1\right]$	Permanent
4.	$\left[G\ \frac{(41,\ 10)}{15}, 3\right]$	Temporary
5.	$\left[G\ \frac{(41,10)}{10} + G\ \frac{(48,\ 18)}{28}, 4\right] = \left[G\ \frac{(89,\ 28)}{43}, 4\right]$	Temporary

Node 2's temporary label $[G\frac{(95, 15)}{15}, 1]$ obtained in iteration 1 is changed to $[G\frac{(55, 14)}{23}, 4]$, node 2 yields the smaller distance ($u_2 = G\frac{55, 14)}{23}$) in iteration 3 to indicate that a shorter route has been found through node 4, the list for iteration 3 shows that the label for node 2 is now permanent. Also node 5's temporary label $[G\frac{(90, 5)}{10}, 3]$ obtained in iteration 2 is changed to $[G\frac{(89, 28)}{43}, 4]$ see Figure 16, node 5 yields the smaller distance ($u_5\ G\frac{(89, 28)}{43} =$).

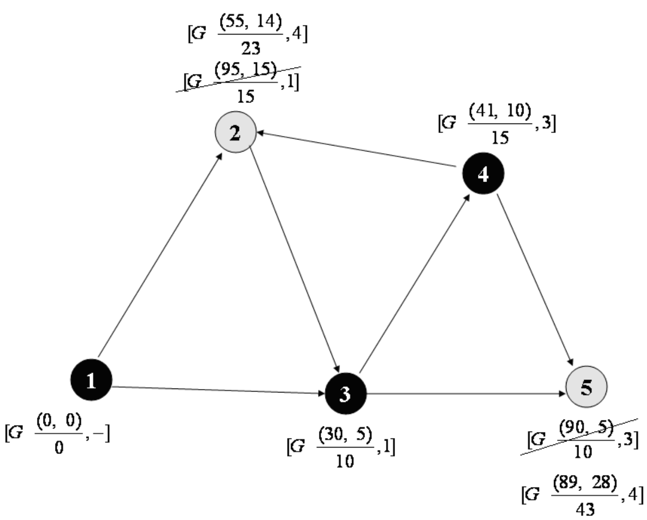

Fig. 16: Iteration 3

Iteration 4:

The list for iteration 3 shows that the label for node 2 is now permanent. Only node 3 can be reached from node 2. However, node 3 has a permanent label and cannot be relabeled. The new list of labels remains the same as in iteration 3 except that the label at node 2 is now permanent as illustrated in TABLEIV.

This leaves node 5 as the only temporary label see Figure 17. Because node 5 does not lead to other nodes, its status is converted to permanent, and the process end.

The computations of the algorithm can be carried out more easily on the network as figures 12, 13, 14, 15, 16 and 17 demonstrate

Table 4 : Iteration 4

Node	Lable	Status
1.	$[G\frac{(0,0)}{0}, -]$	Permanent
2.	$[G\frac{(55, 14)}{23}, 4]$	Temporary
3.	$[G\frac{(30, 5)}{10}, 1]$	Permanent
4.	$[G\frac{(41, 10)}{15}, 3]$	Temporary
5.	$[G\frac{(89, 28)}{43}, 4]$	Temporary

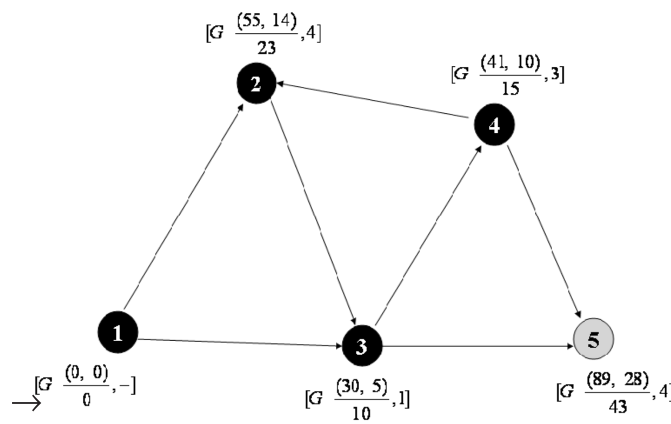

Fig. 17 : Iteration 4

The shortest route between nodes 1 and any other node in the network is determined by starting at the desired destination node and backtracking through the nodes using the information given by the permanent labels. For example, the following sequence determines the shortest route from node 1 to node 2:

(2) $[G\frac{(55, 14)}{23}, 4]$ (4) $\rightarrow$ $[G\frac{(41, 10)}{15}, 3]$ (3) $\rightarrow$ $[G\frac{(30, 10)}{15}, 1]$ $\rightarrow$ (1)

Thus, the desired route is $1 \rightarrow 3 \rightarrow 4 \rightarrow 2$ with a total length of $G\frac{(55, 14)}{23}$ which is around 55, this result is the same as the one in classical case.

VI. CONCLUSION

Unified granular number or G-number is a general representation for different types of uncertain numbers.

The main benefit from using G-number in uncertain optimization problems is providing the ability to homogenate the manipulation of heterogeneous types of uncertain numbers via converting each of them into the new representation of G-number, then the same set of arithmetic calculations rules can be utilized. It is worth to mention that, the implemented approaches are general ones, in which the fuzzy, rough, interval or crisp classical solutions are considered as special cases. As a future work we plan to develop a ranking method for G-number according to its parameters also we will work for transforming more types of uncertain numbers such as: vague number, grey number and type-2 fuzzy number into the general form of G-number.

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