Deformation in Transversely Isotropic Thermoelastic Material Without Energy Dissipation and With Two Temperature due to Inclined Load

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Abstract: *The present investigation is concerned with the two dimensional deformation in a homogeneous, transversely isotropic thermoelastic solids without energy dissipation and with two temperature as a result of an inclined load. The inclined load is assumed to be linear combination of normal load and tangential load. Laplace and Fourier transforms are used to solve the problem. The components of displacements, stresses and conductive temperature distribution so obtained in the physical domain are computed numerically. Effect of two temperature are depicted graphically on the resulting quantites.*

Key words: *Two temperature, without energy dissipation, transversely isotropic thermoelastic, Laplace transform, Fourier transform, concentrated and distributed sources*

I. INTRODUCTION

Thermoelasticity is the study of interaction between deformation and thermal fields. It deals with dynamical system whose interaction with surroundings is limited to mechanical work, external forces and heat exchange. It also comprises the heat conduction, stress and strain that arise due to flow of heat. Also, the change of body temperature is caused not only by external and internal heat sources but by a process of deformation itself. For this reason, thermoelasticity is to be regarded as a multi-field discipline, governed by the interaction of a temperature deformation field. It makes possible to determine the stresses produced by the temperature field and to calculate the temperature distribution due to an action of time dependent forces and heat sources.

Green and Naghdi [5] and [6] postulated a new concept in generalized thermoelasticity and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical Thermoelastic model. In model -II, the internal rate of production entropy is taken to be identically zero implying no dissipation of thermal energy . This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as theory of thermoelasticity without energy dissipation. The principal feature of this theory is in contrast to classical thermoelasticity associated with Fourier's law of heat conduction, the heat flow does not involve energy dissipation. This theory permits the transmission of heat as thermal waves at finite speed. Model-III includes the previous two models as special cases and admits dissipation of energy in general. In context of Green and Naghdi model many applications have been found. Chandrasekharaiah and Srinath [1] discussed the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity. Kumar and Deswal [8] studied the surface wave propagation in a micropolar thermoelastic medium without energy dissipation.

Youssef [16] constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Chen and Gurtin [2], Chen et al.[3] and [4] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body. Warren and Chen [12] investigated the wave propagation in the two temperature theory of thermoelasticity. Quintanilla [11] proved some theorems in thermoelasticity with two temperatures. Youssef AI-Lehaibi [14] and Youssef AI - Harby [15] investigated various problems on the basis of two temperature thermoelasticity. Kaushal, Kumar and Miglani [7] discussed response of frequency domain in generalized thermoelasticity with two temperatures. Sharma and Kumar [9-10] discussed elastodynamic response and interactions of generalised thermoelastic diffusion due to inclined load.

In this paper, a general solution has been obtained in the transformed form using the Laplace and Fourier transforms to the field equations of a transversely isotropic thermoelastic without energy dissipation and with two temperature due to various sources. Concentrated source have been taken to illustrate the utility of the approach as an application. Numerical inversion technique is applied to invert numerically the transformed solutions. The results in the form of displacement components, conductive temperature and stress components have been obtained numerically and illustrated graphically for particular model.

II. BASIC EQUATIONS

Following H.M.Youssef [16] the constitutive relations and field equations are:

$$t_{ij} = C_{ijkl}\, e_{kl} - \beta_{ij}\, T \qquad (1)$$

$$C_{ijkl}\, e_{kl,j} - \beta_{ij} T_j + \rho F_i = p\ddot{u}_\iota \qquad (2)$$

$$K_{ij}\, \varphi,_{ij} = \beta_{ij}\, T_o \ddot{e}_{ij} + \rho C_E T \qquad (3)$$

Where

$$T = \varphi - \alpha_{i,j}\varphi_{ij} \qquad \beta_{ij} = C_{ijkl}\, \alpha_{ij}$$

$$e_{ij} = u_{i,j} + u_{j,i} \qquad i,j = 1,2,3$$

Here

C_{ijkl} ($C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$) are elastic parameters, β_{ij} is the thermal tensor, T is the temperature, T_o is the reference temperature, t_{ij} are the components of stress tensor, e_{ki} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the thermal conductivity, α_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

Applying the transformation

$$x'_1 = x_1\cos\theta + x_2 \sin\theta,\ x'_2 = -x_1 \sin\theta + x_2 \cos\theta,\ x'_3 = x_3 \text{ where } \theta \text{ is the angle of rotation in } x_1 - x_2 \text{ plane} \qquad (4)$$

The basic equations reduce to

$$c_{11}u_{1,11} + c_{12}u_{2,21} + c_{13}u_{3,31} + c_{66}\left(u_{1,22} + u_{2,12}\right) + c_{44}\left(u_{1,33} + u_{3,13}\right) - \beta_1 \frac{\delta}{\delta x_1}\left\{\varphi - \left(a_1\, \varphi_{,11} + a_2\, \varphi,_{22} + a_3 d,_{33}\right)\right\} + \rho F_1 = \rho\ddot{u}_1 \qquad (5)$$

$$c_{11}\ \ u_1,_{12} + u_{2,22} + c_{66}u_{2,11} + c_{44}u_{2,23} + \left(c_{13} + c_{44}\right) u_{3,32} - \beta_2 \frac{\delta}{\delta x_2}\left\{ \varphi - \left(a_1\varphi_{,11} + a_2\varphi_{,22} + a_3\varphi_{,33}\right)\right\} + \rho F_2 = \rho\ddot{u}_2 \qquad (6)$$

$$\left(c_{13} + c_{44}\right)\left(u_{1,13} + u_{2,23}\right) + c_{44}\left(u_{3,11} + u_{3,22}\right) + c_{33}u_{3,33} - \beta \frac{\delta}{\delta x_3}\left\{ \varphi - \left(a1,\varphi_{,11} + a_2\, \varphi_{,22} + a_3\varphi_{,33}\right)\right\} \rho F_3 = \rho\ddot{u}_3 \qquad (7)$$

$$k_1\varphi,_{11} + k_2\varphi,_{22} + k_3\varphi_{,33} = T_0\left(\beta_1\ddot{e}_{11} + \beta_2\ddot{e}_{22} + \beta_3\ddot{e}_{33}\right) + \rho C_E\left\{\ddot{\varphi} - \left(a1\, \ddot{\varphi},_{11} + a_2\ddot{\varphi},_{22} a_3\ddot{\varphi},_{33}\right)\right\} \qquad (8)$$

In the above equations we use the contracting subscript notations (1 → 11,2 → 22,3 → 33,5 → 23,4 → 13,6 → 12) to relate c_{ijkl} to c_{mn}.

III. PROBLEM FORMULATION

We consider a homogeneous, transversely isotropic thermoelastic solid half-space with two temperatures. We take rectangular Cartesian co-ordinate *system* (x_1, x_2, x_3) having origin on the surface $x_3 = 0$ with x_3 axis pointing vertically downwards into the half-space. Suppose an inclined load F_0,per unit length is acting on the x_2 axis and its inclination with x_3 axis is δ.

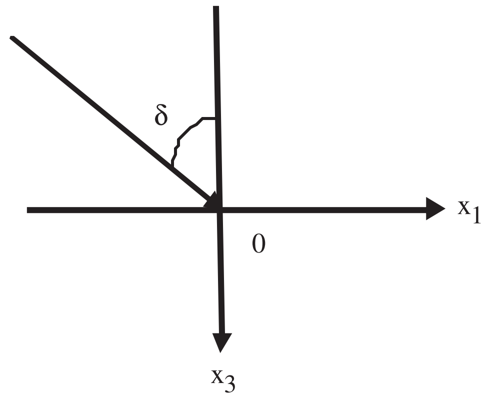

Fig.1: Inclined load over a thermoelastic solid

We restrict our analysis in two dimensions subject to plane parallel to x_1 - x_3 plane. The displacement vector for two dimensional problems is taken as :

$$u = (u_1, 0, u_3) \tag{9}$$

The basic governing equations (2) and (3) using (9) and in absence of body forces are given as:

$$c_{11}u_{1,11} + c_{13}u_{3,31} + c_{44}(u_{1,33} + u_{3,13}) - \beta_1 \frac{\delta}{\delta x_1}\{\varphi - (a_1\varphi,_{11} + a_3\varphi,_{33})\} = \rho\ddot{u}_1 \tag{10}$$

$$(c_{13} + c_{44})\, u_{1,13} + c_{44}u_{3,11} + c_{33}u_{3,33} - \beta_3 \frac{\delta}{\delta x_3}\{\varphi - (a_1,\varphi,_{11} + a_3\varphi,_{33})\} = \rho\ddot{u}_3 \tag{11}$$

$$k_1\varphi,_{11} + k_3\varphi,_{33} = T_0(\beta_1\ddot{e}_{11} + \beta_3\ddot{e}_{33}) + \rho C_E\{\ddot{\varphi} - (a_1\ddot{\varphi},_{11} + a_3\ddot{\varphi},_{33})\} \tag{12}$$

where $\beta_1 = c_{11}\alpha_1 + c_{31}\alpha_3$, $\beta_3 = c_{31}\alpha_1 + c_{33}\alpha_3$

To facilitate the solution, following dimensionless quantities are introduced:

$$x'_1 = \frac{x_1}{L},\ x'_3 = \frac{x_3}{L},\ u'_1 = \frac{\rho c_1^2}{L\beta_1 T_0}u_1,\ u'_3 = \frac{\rho c_1^2}{L\beta_1 T_0}u_3,$$
$$T' = \frac{T}{T_0},\ t' = \frac{c_1}{L}t,\ t'_{11} = \frac{t_{11}}{\beta_1 T_0},\ t'_{33} = \frac{t_{33}}{\beta_1 T_0},$$
$$t'_{31} = \frac{t_{31}}{\beta_1 T_0},\ \varphi' = \frac{\varphi}{T_0},\ a'_1 = \frac{a_1}{L},\ a'_3 = \frac{a_3}{L} \tag{13}$$

Where $C_1^2 = \frac{c_{11}}{\rho}$ and L is a constant of dimension of length.

Using the dimensionless quantities defined by (13) into (10) - (12) and after that suppressing the primes we obtain.

$$\frac{\delta^2 u_1}{\delta x_1^2} + \delta_1\frac{\delta^2 u_1}{\delta x_3^2} + \delta_2\frac{\delta^2 u_3}{\delta x_1\delta x_3} - \left[1-\left(a_1\frac{\delta^2}{\delta x_1^2} + a_3\frac{\delta^2}{\delta x_3^2}\right)\right]\frac{\delta\varphi}{\delta x_1} = \frac{\delta^2 u_1}{\delta t^2} \tag{14}$$

$$\delta_4\frac{\delta^2 u_3}{\delta x_3^2} + \delta_1\frac{\delta^2 u_3}{\delta x_1^2} + \delta_2\frac{\delta^2 u_1}{\delta x_1\delta x_3} - p_5\left[1-\left(a_1\frac{\delta^2}{\delta x_1^2} + a_3\frac{\delta^2}{\delta x_3^2}\right)\right]\frac{\delta\varphi}{\delta x_3} = \frac{\delta^2 u_3}{\delta t^2} \tag{15}$$

$$\frac{\delta^2\varphi}{\delta x_1^2} + p_3\frac{\delta^2\varphi}{\delta x_3^2} - \zeta_1\frac{\delta^2}{\delta t^2}\frac{\delta u_1}{\delta x_1} - \zeta_2\frac{\delta^2}{\delta t^2}\frac{\delta u_3}{\delta x_3} = \zeta_3\left[1-\left(a_1\frac{\delta^2}{\delta x_1^2} + a_3\frac{\delta^2}{\delta x_3^2}\right)\right]\frac{\delta^2\varphi}{\delta t^2} \tag{16}$$

where

$$\delta_1 = \frac{c_{44}}{c_{11}},\ \delta_2 = \frac{c_{13} + c_{44}}{c_{11}},\ \delta_4 = \frac{c_{33}}{c_{11}},\ = p_5\ \frac{\beta_3}{\beta_1},\ p_3 = \frac{k_3}{k_1},\ \zeta_1 = \frac{T_0\beta_1^2}{k_1\rho},\ \zeta_2 = \frac{T_0\beta_3\beta_1}{k_1\rho},\ \zeta_3 = \frac{C_E c_{11}}{k_1} \tag{17}$$

Apply Laplace and Fourier transforms defined by

$$\overline{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t)\, e^{-st}\, dt$$
$$\hat{f}(\zeta, x_3, s) = \int_{-\infty}^{\infty} f(x_1, x_3, s) e^{i\zeta, x1} dx_1 \tag{18}$$

on equations (14) - (16) and then eliminating u_1, u_3 and φ we obtain

$$\left(P\frac{d^6}{dx_3^6} + Q\frac{d^4}{dx_3^4} + R\frac{d^2}{dx_3^2} + S\right)(u_1, \hat{u}_3, \hat{\varphi}) = 0 \tag{19}$$

Where $P = \delta_1(\delta_4\zeta_3 a_3 s_2 - \delta_4 p_3 - \zeta_2 p_5 a_3 s^2)$

$Q = (\zeta_3 a_3 s^2 - p_3)\{(-\xi^2 + s^2)\delta_4 - \delta_1(b_1\xi^2 + s^2) + \delta_2^2\xi^2\} + \delta_1\delta_4\{\xi^2 - \zeta_3 s^2 - \xi^2\zeta_3 s^2 a_1\} + \zeta_2 s^2\{a_3 p_5(\xi^2 + s^2) + \delta_1 p_5(a_1\xi^2 + 1)\} + \xi^2 s^2\{-\delta_4 a_3(p_5\zeta_1 + \zeta_2 - \zeta_1)\}$

$R = (1 + a_1\xi^2)\{-(\xi^2 + s^2)\zeta_2 p_5 s^2 + \xi^2 s^2(p_5\zeta_1\delta_2 + \zeta_2\delta_2 - \zeta_1\delta_4)\} + (\delta_1\xi^2 + s^2)\{\xi^2 + s^2)(s^2\zeta_3 a_3 - p_3) - \delta_1(\xi_2 - \zeta_3 s^2 - \zeta_3 s^2 a_1\xi^2) - \xi^2 a_3\zeta_1 s^2\} + (\xi^2 - \zeta_3 s^2 - \zeta_3 s^2\xi^2 a_1)\{-(\xi^2 + s^2)\delta_4 + \delta_2^2\xi^2\}$

$$S = (\delta_1 \xi^2 + s^2) \{(\xi^2 + s^2)(\xi^2 - \zeta_3 s^2 - \zeta_3 s^2 a_1 \xi^2) + \xi^2 (1 + a_1 \xi^2) \xi^2 \zeta_1 s^2\} \quad (20)$$

The roots of the equation are $\pm \lambda_i$, i = 1, 2, 3. Making use of the radiation conditions that $\hat{u}_1, \hat{u}_3, \hat{\varphi} \to 0$ as $x_3 \to \infty$ the solution of the equation (19) may be written as :

$$\hat{u}_1 = A_1 e^{-\lambda 1x3} + A_2 e^{-\lambda 2x3} + A_3 e^{-\lambda 3x3}$$

$$\hat{u}_3 = d_1 A_1 e^{-\lambda 1x3} + d_2 A_2 e^{-\lambda 2x3} + d_3 A_3 e^{-\lambda 3x3}$$

$$\hat{\varphi} = d'_1 A_1 e^{-\lambda 1x3} + d'_2 A_2 e^{-\lambda 2x3} + d'_3 A_3 e^{-\lambda 3x3} \quad (21)$$

Where

$$d_i = \frac{-\lambda_i^3 P^* - \lambda_i Q^*}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1, 2, 3$$

$$li = \frac{-\lambda_i^2 P^{**} - Q^{**}}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1, 2, 3 \quad (22)$$

Where $P^* = i\zeta \{(-\zeta_1 p_5 a_3 s^2 + \delta_2 (\zeta_3 a_3 s^2 - p_3)\}$

$$Q^* = \delta_2 (\xi^2 - \zeta_3 s^2 - \zeta_3 s^2 a_1 \xi^2) + p_5 \zeta_1 (1 + a_1 \xi^2) s^2$$

$$R^* = -\zeta_2 p_5 a_3 s^2 + \delta_4 (\zeta_3 a_3 s^2 - p_3)$$

$$S^* = (\xi^2 - \zeta_3 s^2 - \zeta_3 s^2 a_1 \xi^2) \delta_4 - (\delta_1 \xi^2 + s^2)(a_3 \zeta_3 s^2 - p_3) + \zeta_2 p_5 s^2 (1 + a_1 \xi^2)$$

$$T^* = -(\delta_1 \xi^2 + s^2)(\xi^2 - \zeta_3 s^2 - \zeta_3 s^2 a_1 \xi^2)$$

$$P^{**} = (\zeta_2 \delta_2 - \zeta_1 \delta_4) s^2 i\zeta$$

$$Q^{**} = \zeta_1 s^2 (\delta_1 \xi^2 + s^2) \quad (23)$$

IV. BOUNDARY CONDITIONS

We consider a normal line load F_1 acting in the positive x_3 axis on the plane boundary $x_3 = 0$ along the x_2 axis and a tangential load F_2, per unit length acting at the origin in the positive x_1 axis. The boundary conditions on the surface $x_3 = 0$ are:

(1) $t_{33} = -F_1 \psi_1 (x) H (t)$

(2) $t_{31} = -F_2 \psi_2 (x) H (t)$

3 $\dfrac{\delta \varphi}{\delta x_3} = 0$ (24)

Where F_1 and F_2 are the magnitudes of the forces applied, $\psi_1 (x)$, $\psi_2 (x)$ specify the vertical and horizontal local distribution functions along x_1 axis, H (t) is the Heaviside unit step function.

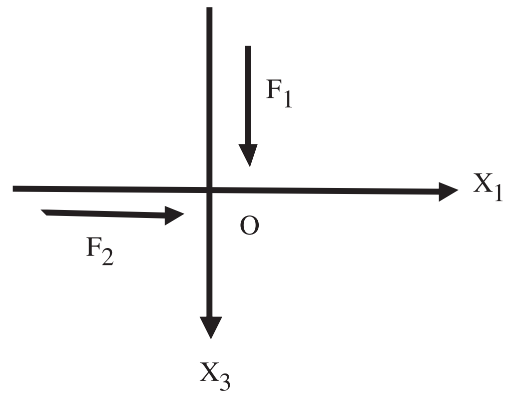

Fig. 2: Normal and tangential loadings

Using the dimensionless quantities defined by (13) on (24) and then applying Laplace Transform and Fourier Transform defined by (18) we obtain

$$\hat{u}_1 = \frac{F_1 \hat{\psi}_1(\xi)}{s\ \Delta} (-M_{11} + M_{12} - M_{13}) + \frac{F_2 \hat{\psi}_2(\xi)}{s\ \Delta} (M_{21} - M_{22} + M_{23})$$

$$\hat{u}_3 = \frac{F_1 \hat{\psi}_1(\xi)}{s\ \Delta} (-d_1 M_{11} + d_2 M_{12} - d_3 M_{13}) + \frac{F_2 \hat{\psi}_2(\xi)}{s\ \Delta} (l_1 M_{21} - d_2 M_{22} + d_3 M_{23})$$

$$\hat{\varphi} = \frac{F_1 \hat{\psi}_1(\xi)}{s\ \Delta} (-1_1 M_{11} + l_2 M_{12} - l_3 M_{13}) + \frac{F_2 \hat{\psi}_2(\xi)}{s\ \Delta} (h_1 M_{21} - l_2 M_{22} + l_3 M_{23})$$

$$\hat{t}_{33} = \frac{F_1 \hat{\psi}_1(\xi)}{s\ \Delta} (-h_1 M_{11} + h_2 M_{12} - h_3 M_{13}) + \frac{F_2 \hat{\psi}_2(\xi)}{s\ \Delta} (h_1 M_{21} - h_2 M_{22} + h_3 M_{23})$$

Where

$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}$, $M_{12} = \Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{32}$,
$M_{13} = \Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}$, $M_{21} = \Delta_{12}\Delta_{33} - \Delta_{13}\Delta_{22}$,
$M_{22} = \Delta_{11}\Delta_{33} - \Delta_{13}\Delta_{31}$, $M_{23} = \Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}$

$$\Delta_{1i} = \frac{c_{31}}{\rho c_1^2} i\xi - \frac{c_{33}}{\rho c_1^2} d_i \lambda_i - \frac{\beta_3}{\beta_1} l_i + \frac{\beta_3}{\beta_1 T_0} l_i \lambda_i^2 \quad i = 1, 2, 3$$

$$\Delta_{2i} = -\frac{c_{44}}{\rho c_1^2} \lambda_i + \frac{c_{44}}{\rho c_1^2} i\xi d_i \quad i = 1, 2, 3$$

$$\Delta_{3i} = \lambda_i l_i \quad i = 1, 2, 3$$

$$\Delta = \Delta_{11} M_{11} - \Delta_{12} M_{12} + \Delta_{13} M_{13}$$

$$h_i = \frac{c_{31}}{\rho c_1^2} i\xi - \frac{c_{33}}{\rho c_1^2} d_i\lambda_i - \frac{\beta_3}{\beta_1} l_i + \frac{\beta_3}{\beta_1 T_0} l_i\lambda_i^2$$

$i = 1, 2, 3$

$$h'_i = -\frac{c_{44}}{\rho c_1^2}\lambda_i + \frac{c_{44}}{\rho c_1^2} i\xi d_i \qquad i = 1, 2, 3$$

Case (i). Concentrated force:

The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x) = \delta(x) \tag{26}$$

Applying Laplace and Fourier transform defined by (18) on (26), we obtain $\widehat{\psi_1(\xi)} = 1$. Using (24) and (25) we obtain the components of displacement, stress and conductive temperature.

$$\hat{\hat{u}}_1 = \frac{F_1}{s\Delta}(-M_{11} + M_{12} - M_{13})$$

$$\hat{\hat{u}}_3 = \frac{F_1}{s\Delta}(-d_1 M_{11} + d_2 M_{12} - d_3 M_{13})$$

$$\hat{\hat{\varphi}} = \frac{F_1}{s\Delta}(-l_1 M_{11} + l_2 M_{12} - l_3 M_{13})$$

$$\hat{\hat{t}}_{33} = \frac{F_1}{s\Delta}(-h_1 M_{11} + h_2 M_{12} - h_3 M_{13})$$

$$\hat{\hat{t}}_{31} = \frac{F_1}{s\Delta}(-h'_1 M_{11} + h_2 M_{12} - h'_3 M_{13})$$

Case (ii). Uniformly distributed force:

Solution due to uniformly distributed force applied on the half space is obtained by setting.

$$\psi_1(x) = \begin{cases} 1 \text{ if } |x| \le a \\ 0 \text{ if } |x| \ge a \end{cases} \tag{27}$$

In equations and using (13), (18), (24) we obtain

$$\widehat{\psi_1(\xi)} = \left[2\sin(\xi a)/\xi\right] \qquad \xi \ne 0$$

using (27), we can obtain components of displacement, stress and conductive temperature.

Case (iii). Linearly distributed force:

Solution due to linearly distributed force applied on the half space is obtained by setting

$$\psi_1(x) = \begin{cases} 1 - \frac{|x|}{a} & \text{if } |x| \le a \\ & 0 \text{ if } |x| \ge a \end{cases}$$

In equations and using (13), (18), (24) we obtain

$$\widehat{\psi_1(\xi)} = \left[2\sin(\xi a)/\xi\right] \qquad \xi \ne 0$$

using (27), we can obtain components of displacement, stress and conductive temperature.

V. APPLICATIONS

A. Suppose an inclined load F_0, per unit length is acting on the x_2 axis and its inclination with x_3 axis is

$$F_1 = F_0 \cos\delta$$

$$F_2 = F_0 \sin\delta$$

In this case, we obtain the expressions for displacements, temperature distribution and stresses in thermoelastic half space using (25) as

$$\hat{\hat{u}}_1 = \frac{F_0\cos\delta\widehat{\psi}_1(\xi)}{s\,\Delta}(-M_{11} + M_{12} - M_{13}) + \frac{F_0\sin\delta\widehat{\psi}_2\xi)}{s\,\Delta}(M_{21} - M_{22} + M_{23})$$

$$\hat{\hat{u}}_3 = \frac{F_0\cos\delta\widehat{\psi}_1(\xi)}{s\,\Delta}(-d_1 M_{11} + d_2 M_{12} - d_3 M_{13}) + \frac{F_0\sin\delta\widehat{\psi}_2(\xi)}{s\,\Delta}(d_1 M_{21} - d_2 M_{22} + d_3 M_{23})$$

$$\hat{\hat{\varphi}} = \frac{F_0\cos\delta\widehat{\psi}_1(\xi)}{s\,\Delta}(-l_1 M_{11} + l_2 M_{12} - l_3 M_{13}) + \frac{F_0\sin\delta\widehat{\psi}_2(\xi)}{s\,\Delta}(l_1 M_{21} - l_2 M_{22} + l_3 M_{23})$$

$$\hat{\hat{t}}_{33} = \frac{F_0\cos\delta\widehat{\psi}_1(\xi)}{s\,\Delta}(-h_1 M_{11} + h_2 M_{12} - h_3 M_{13}) + \frac{F_0\sin\delta\widehat{\psi}_2(\xi)}{s\,\Delta}(h_1 M_{21} - h_2 M_{22} + h_3 M_{23})$$

$$\hat{\hat{t}}_{31} = \frac{F_0\cos\delta\widehat{\psi}_1(\xi)}{s\,\Delta}(-h'_1 M_{11} + h'_2 M_{12} - h'_3 M_{13}) + \frac{F_0\sin\delta\widehat{\psi}_2(\xi)}{s\,\Delta}(h'_1 M_{21} - h'_2 M_{22} + h'_3 M_{23})$$

B. Particular case

In case of isotropic elastic solid, we have

$$c_{11} = \lambda + 2\mu = c_{33},\ c_{12} = c_{13} = \lambda,\ c_{44} = \mu$$

C. Inversion of the Transformation

The transformed stresses and temperature distribution are functions of ξ, x_3 and s the parameters of Laplace and Fourier transforms and hence are of the form $f(\xi, x_3, s)$. The obtain the solution of the problem in the physical domain, we first invert the Fourier transform using

$$\overline{f}(x_1, x_3, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x_1} f(\xi, x_3, s)\, d\xi, = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mid \cos(\xi x_1) f_e - i\sin(xx1) f_0 \mid d\xi \quad (51)$$

Where f_0 and f_e are respectively the odd and even parts of $f(\xi, x_3, s)$. thus the above expression gives the Laplace transform $f(x_1, x_3, s)$ of the function $f(x_1, x_3, t)$:

VI. RESULTS AND DISCUSSION

Copper material is chosen for the purpose of numerical calculation which is transversely isotropic.

$c_{11} = 18.78 \times 10^{10}$ $Kgm^{-1}s^{-2}$,
$c_{12} = 8.76 \times 10^{10}$ $Kgm^{-1}s^{-2}$,
$c_{13} = 8.0 \times 10^{10}$ $Kgm^{-1}s^{-2}$

$C_{33} = 17.2 \times 10^{10}$ $Kgm^{-1}s^{-2}$,
$c_{44} = 5.06 \times 10^{10}$ $Kgm^{-1}s^{-2}$,
$C_E = 0.6331 \times 10^3$ $JKg^{-1}K^{-1}$

$\alpha_1 = 2.98 \times 10^{-5}$ K^{-1}, $\alpha_3 = 2.4 \times 10^{-5}$ K^{-1},
$a = 2.4 \times 10^4$ m^2 s^{-2}, $b = 13 \times 10^5$ m^5 s^{-2} Kg^{-1}

$\rho = 8.954 \times 10^3$ Kgm^{-3}, $K_1 = 0.433 \times 10^3$ $Wm^{-1}K^{-1}$,
$K_3 = 0.450 \times 10^3$ $Wm^{-1}K^{-1}$

The values of normal displacement u_3, normal force stress t_{33}, tangential stress t_{31} for a transversely isotropic thermoelastic solid at $\theta = 0°$, $\theta = 45°$ i.e. at initial angle, middle angle and extreme angle have been sudied. The variations of these components due to impulsive load with distance x and for two temperatures a=0 and a=.05 have been shown in the Fig. 1 to 4.

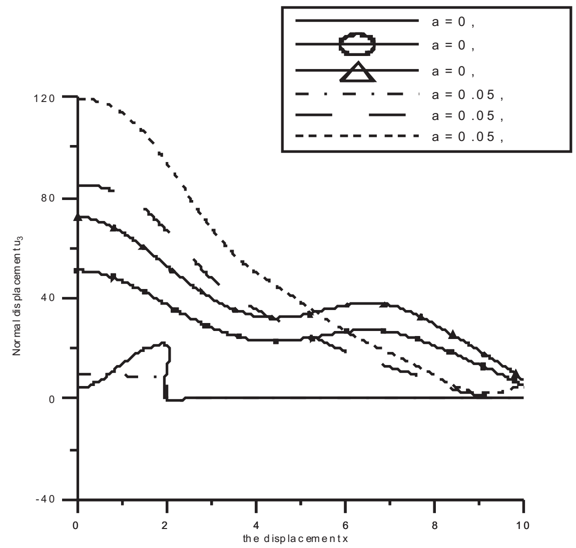

Fig. 1: Variation of normal displacement u_3 with distance x

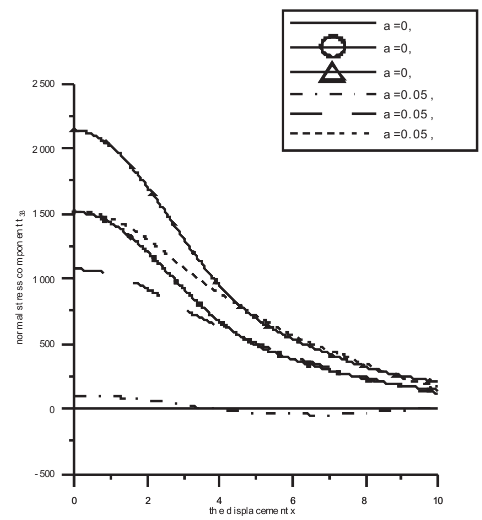

Fig. 2: Variation of normal stress component t_{33} with distance x

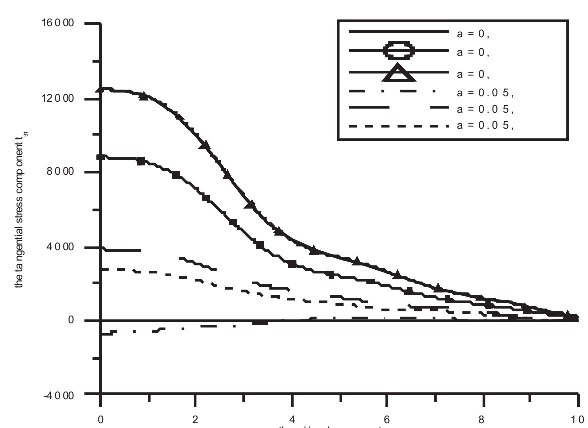

Fig. 3: Variation of tangential stress component t_{31} with displacement x

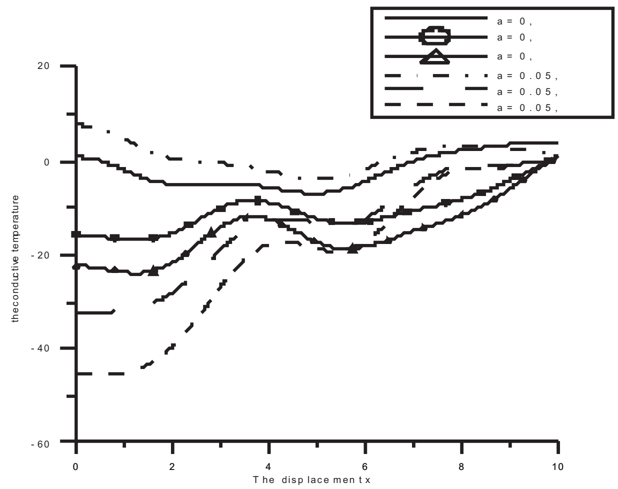

Fig. 4: Variation of conductive temperatue ϕ with displacement x

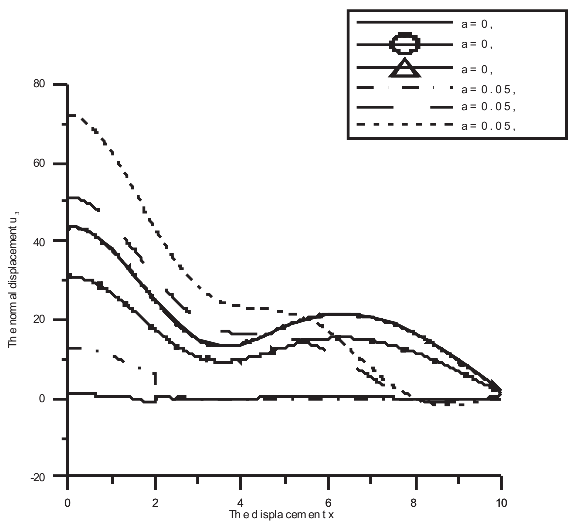

Fig. 5: Variation of normal displacement u_3 with distance x due to Linearly distributed load.

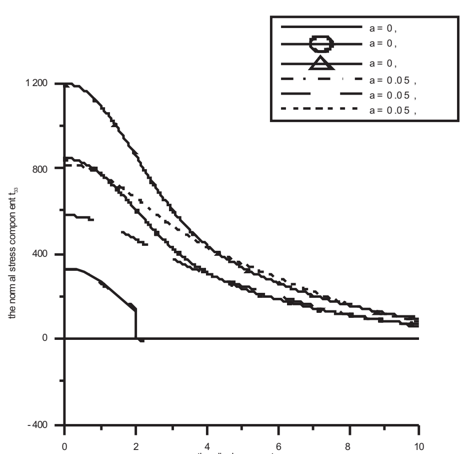

Fig. 6: Variation of normal stress component t_{33} with displacement x

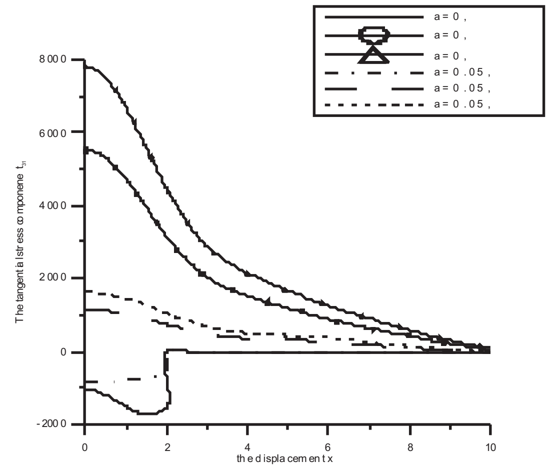

Fig. 7: Variation of tangential stress component t_{31} displacement x.

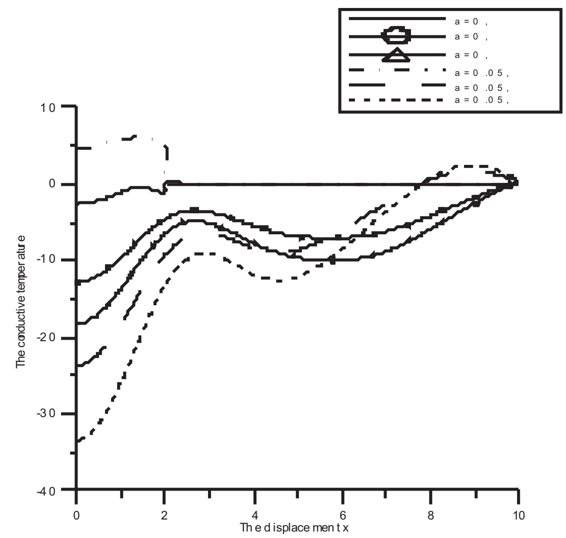

Fig. 8: Variation of conductive temperatue ö with displacement x

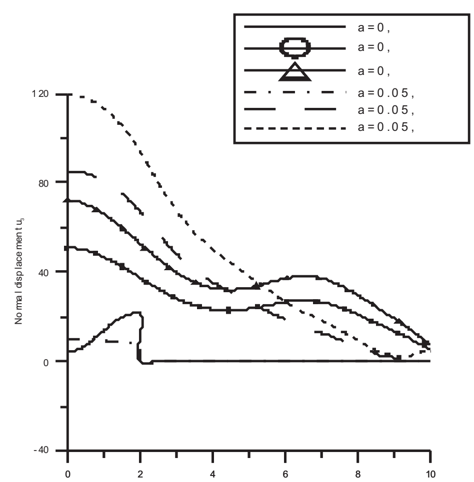

Fig. 9: Variation of normal displacement u_3 with distance x due to Linearly distributed load.

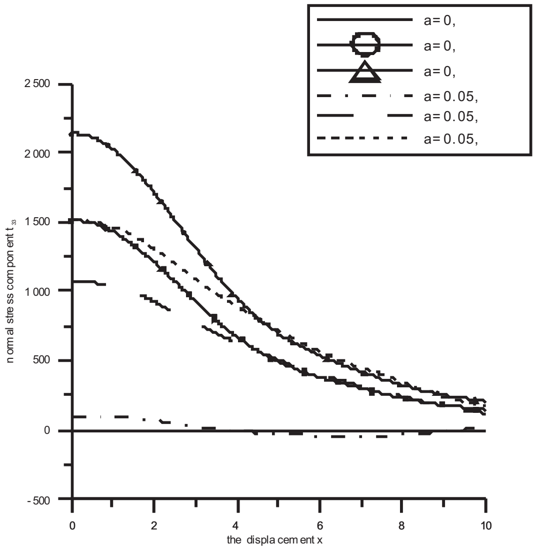

Fig. 10: Variation of normal stress component t_{33} with displacement x

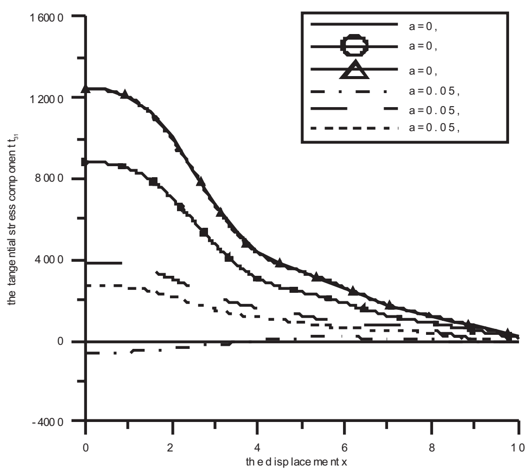

Fig. 11: Variation of tangential stress component t_{31} displacement x.

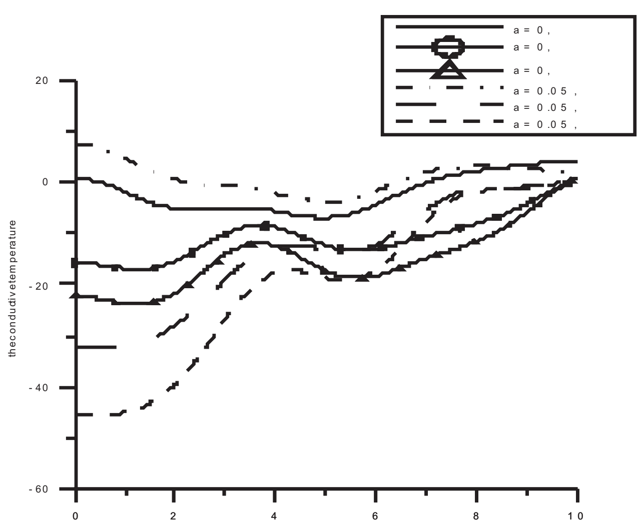

Fig. 12: Variation of conductive temperatue ö with displacement x

1. The solid line, solid line iwth centre symbol circle, solid line with centre symbol triangle respectively predict the variations at a = 0 and at angle of inclination θ = 0°, q = 45° and θ.

2. The dash dot dash line, the long dashed line, the small dashed line respectively predict the variations at a = 0.05 and at angle of inclination θ = 0°, q = 45° and θ.

VII. CONCLUSION

Effect of two temperature have significant impact on components of normal displacement , normal stress, tangential stress and conductive temperature. As disturbance travels through the constituents of the medium, it suffers sudden changes resulting in an inconsistent / non uniform pattern of graphs. The deformation in any part of the medium is useful to analyse the deformation field around mining tremors and drilling into the crust. It can also contribute to the theoretical consideration of the seismic and volcanic sources. Since it can account for the deformation fields in the entire volume surrounding the sources region.

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