A note on Chang's and Lowen's Fuzzy Topology

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Abstract: *In this paper, we establish equivalence between Chang's and Lowen's Fuzzy topological spaces.*

I. INTRODUCTION

Topology is an important branch of mathematics which concern with properties that are preserved under continuous deformations of objects, such as deformations that involve stretching, but no tearing or gluing. Study of topology began with the Euler's problem of Seven Bridges of Konigsberg, which is regarded as one of the first academic records in modern topology, as Euler was searching a solution to a geometrical problem where distance has no relevance. Modern topology is based on the ideas of set theory. The study of Fuzzy topology as a generalization of crisp topology was initiated by C.L. Chang [1] in 1968 after Zadeh's [6] introduction of fuzzy set theory in the year 1965. Lowen pointed out the antinomies of Chang's fuzzy topology and established the need by a new definition of fuzzy topological space. A. Sostak [5] and Hazra et. al [2] argue the lack of fuzziness in both of the above definitions of fuzzy topology. They came up (separately) with new definitions of fuzzy topology in true fuzzy sense. In the area of fuzzy topology much work has been done round the globe since its introduction.

In this paper, we are to establish the equivalence between the fuzzy topologies in the sense of Chang and that of Lowen.

II. PRELIMINARIES

Below are some definitions required in the sequel. We begin with the definition of a fuzzy set.

Definition 2.1. A Fuzzy set on a set X is a pair (X; A), where A is a membership function given by A: $X \to [0,1]$. For each $x \in X$, A (x) is called the grade of membership or the membership value of x in the fuzzy set A. If I = [0, 1], then I^X denotes the set of all I-valued fuzzy sets on X.

Throughout this note, by FTS we mean fuzzy topological space.

Definition 2.2. [1] A fuzzy topological space is a pair , where X is any set and , satisfying the following axioms:

(i) $0_x, 1_x \in \tau$;

(ii) If $A, B \in \tau$; then $A \wedge B \in \tau$;

(iii) If $A_\lambda \in \tau$ for each , $\lambda \in \Delta$, then $\vee_\lambda A_\lambda \in \tau$.

Here 0_x and 1_x respectively denote the empty set and the whole set.

Definition 2.3. [3] A fuzzy topological space is a pair (X, τ), where X is any set and $\tau \subseteq I^X$; satisfying the following axioms:

(i) $\overline{\alpha} \in \tau$; $\forall\ \overline{\alpha} \in I^X$;

(ii) If $A, B \in \tau$; then $A \wedge B \in \tau$;

(iii) If $A_\lambda \in \tau$ for each $\lambda \in \Delta$, then $V_\lambda A_\lambda \in \tau$.

Here $\overline{\alpha}$ denotes the constant fuzzy set with α as its membership value.

Definition 2.4. [4] A fuzzy set in *X* is called a fuzzy point if and only if it takes the value 0 for all $y \in X$ except one, say $y \in X$. If its value at *x* is λ, $(0 < \lambda \leq)$; then the fuzzy point is denoted by p_x^{λ}.

Let *A* be a fuzzy set in *X*, then $P^{\lambda}_x \subset A \Leftrightarrow \lambda \leq A$ (x). In particular $P^{\lambda}_x P_y \Leftrightarrow x = y, \lambda \leq \gamma$ and $P_x^{\lambda} \in A \Leftrightarrow \lambda < A$ (x).

Definition 2.5. [4] An FTS (X, τ) is said to be a T_1-space if every fuzzy point in *X* is a closed fuzzy set.

Definition 2.6. [1] A function f from an FTS (X, τ) to an FTS (y, δ) is fuzzy continuous if and only if the inverse image of each -open fuzzy set is τ-open.

III. MAIN RESULT

In this section, we establish the conditions under which Chang's FTS and Lowen's FTS become equivalent. As the definitions di_er in one point only, it is suffcient to establish the first condition of Lowen's definition in order to establish the equivalence between the two definitions.

Theorem 3.1. A Chang's FTS in which every constant function is continuous is an FTS in Lowen's sense.

Proof. By definition of fuzzy continuous function, every constant function is open in the FTS. Hence the FTS in

Chang's sense is also a FTS in Lowen's sense.

Theorem 3.2. An FTS (x, τ) in Chang's sense is an FTS in Lowen's sense if it is T_1 and *X* is singleton.

Proof. Let (X, τ) be a Chang's FTS which is T_1 and X = {x}. Since (X, τ) is T_1, so every fuzzy point is closed.

$\Rightarrow$ for every $\alpha \in [0,1]$ $\exists$ $A:X \rightarrow I$ such that $A(x) = \alpha$ and *A* is closed

$\Rightarrow$ for every $\beta \in [0, 1]$, $(\beta = 1 - \alpha)$ $\exists$ $B : X \rightarrow I$ such that $B(x) = \beta = 1 - \alpha$ and *B* is open;

$\Rightarrow$ every constant function in X = {x} is open;

$\Rightarrow$ (X, τ) is an FTS in Lowen's sense.

IV. CONCLUSION

A lot of research work has been done in the field of fuzzy topology. Surprisingly, there is no such result which establishes an equivalence between the definitions of fuzzy topology defined by Chang and that of Lowen. This paper ascertains the conditions under which the two definitions of FTS coincide.

REFERENCES

[1]. Chang, C.L., Fuzzy topological spaces, *J. Math. Anal. Appl.*, 24, (1968), 182–190.

[2]. Hazra, R.N., Samanta, S.K., Chattopadhyay, K.C., Fuzzy topology redefined, *Fuzzy Sets and Systems*, 45, (1991), 78–82.

[3]. Lowen, R., Fuzzy topological spaces and fuzzy compactness, *J. Math. Anal. Appl.*, 56, (1976), 621–633.

[4]. Ming, P., Ming, L., Fuzzy Topology I, Neighborhood Structure of a Fuzzy Point and Moore-Smith Convergence, *J. Math. Anal. Appl.*, 76, (1980), 571–599.

[5]. Sostak, A.P., On a fuzzy topological structure, *Rend. Circ. Mat. Palermo (2) Suppl.*, 11, (1985), 89-103.

[6]. Zadeh, L.A., Fuzzy Sets, *Information and Control,* 11, (1965), 341–356.

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