Evaluation of Realistic and Exact Three Dimensional Finite Element Stresses for Large Excavation

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Abstract: *Modelingof three dimensional excavation using Finite element method were analyzed for major, intermediate, minor stresses as well shear stresses for seven lift. The shear stresses contours were plotted along the vertical excavation corresponding major, intermediate, minor stresses for the each lift of excavation. The seven lift were taken for a depth of 12000 mm. The discretized continuum were taken for the vertical depth of 12000 mm, where as discretization of the physical body was subjected to 24000 mm and 30000 mm. The Vonmises contours for the stresses were plotted for verification of stresses. The meshing and application of equivalent load over the vertical cut were applied was 0.0 70N /mm2 and 4.0 N/mm2. The Element in FEM were taken as quad and the simulation process should be carried out. The Displacement vectors were plotted to visualization of the pattern of the displacement along the vertical plan. The elastic strain for the three dimensional case study were plotted and the plotted Strain diagram, showed the failure mechanism. The mechanical strain for three dimensional contours were plotted to observe the failure surface. The Vonmises's Criteria for assessment of the three dimensional stresses were verified and correlated to the equation for modification. Experimental verification for the three dimensional stresses analysis were examined and verified.*

I. INTRODUCTION

The present problems entitled for Vertical Excavation using finite element method on lime stone and clay materials. Here simulation of Excavation were carried out in sequence for three dimensional. Scope of analysis were to 3D Stresses and Strain, Tension, failure mechanism development for the each element and node.The Problems were subjected to initial loading condition equivalent to .007 N/mm2 and 4.0 N/mm2. All three excavation of the soil had three different loading condition. Hence the each nodes and element were subjected to loading and the failure mechanism corresponding to the application of the load were listed and a comparative statement showing the most critical lift of excavation which would provide the understanding of cracks developed during excavation were examined. Keeping the view of band width of the matrixes and element numbering might be began With the lower part of excavation of the soil. The other way to number the element was to start from the side of the excavation boundary to avoid the complexity of the problem. The coordinate of the each of the element were listed for three dimensional vertical excavations.

II. METHODOLOGY

The boundary condition of the excavation for the case of three dimensional were applied ,however the boundary condition near the bottom layer of the excavation were taken as fixed where the moment as well as forces were existing, but at The corner of the vertical excavation were subjected to fixed supports, and the side of the excavated boundary were as rollers, the above boundary condition were carried out to examine the parameters as mentioned above. The aspect ratio were taken from 1 to 10, however the higher value were subjected to the less effect of stresses, seems to indicate the lower value were subjected to the large variation of the stresses. At the vicinity of the vertical excavation, the aspect ratio were taken as lower 1 to 3. In the case three dimensional vertical excavations, the profile were discretized in the both side of the axis.

A) Approach of FEM for 3Dimensional Vertical Excavation : The domain were discertized in the x-y, y-z and z-x axis, hence the discretized continuum were allocated with their coordinates along the all three axis, the present case analysis were examined with the brick element to reduce the time in formation of band with. The boundary condition of surface were made as in the vertical excavation the horizontal displacement were restricted, however the corners of the surface were restricted for the displacement along the horizontal and vertical direction, the displacement of the bottom layers were also restricted to the displacement as the bottom layers were also fixed, therefore the degree of freedom is constrained during the excavation surface. The all three stresses were analyzed. Summarization in all three plane such as x-y-z were made, final stresses for each element with six shear stresses and three normal stressed were evaluated by means of FEM process.

B) Approach of Experimental verification for Three Dimensional Vertical Excavation stresses: The Triaxial apparatus were taken for the verification of stresses, all three stresses were examined with help of the triaxial apparatus with an partition to apply the pore fluid pressure on the both side of the specimen to examine the vertical or deviatory of major principal stresses in laboratory. The variation of the stresses with the Finite element method three dimensional and Experimental are Examined.

C) **Approach of Finite element modeling with nodal numbering system for Three Dimensional Vertical Excavation stresses**

(i) x-y Plane

168	169	170	171	172	173				177	178	179	180	181	182
152	153	154	155	156	158				162	163	164	165	166	167
137	138	139	140	141	142				146	147	148	149	150	151
122	123	124	125	126	127				131	132	133	134	135	136
106	107	109	110	111	112				116	117	118	119	120	121
91	92	93	94	95	96				100	101	102	103	104	105
76	77	78	79	80	81				85	86	87	88	89	90
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

ii) y-z Plane

336	337	338	339	340					341	342	343	344	345
322	323	324	325	326					331	332	333	334	335
307	308	309	310	312					317	318	319	320	321
293	294	295	296	297					302	303	304	305	306
279	280	281	282	283					288	289	290	291	292
265	266	267	268	269					274	275	276	277	278
251	252	253	254	255					260	261	262	263	264
237	238	239	240	241	242	243	244	245	246	247	248	249	250
223	224	225	226	227	228	229	230	231	232	233	234	235	236
209	210	211	212	213	214	215	216	217	218	219	220	221	222
196	197	197	198	199	200	201	202	203	204	205	206	207	208
183	184	185	186	187	188	189	190	191	192	193	194	195	196

a) Assumption for 3Dimensional Finite Element Method

1 The materials were elastic piece wise linear, Isotropic and homogeneous.

2. Materils Fails essentially by shear. The critical shear stresses were causing failure, depends upon the properties of the materials as well as normal stresses on the failure.

3. The ultimate strength of the materials was determined by the stresses in potential plane (on plane of shear).

4. The modeling were created with all three coordinated x, y, z.

5. The three dimensional, the total number of element were taken as cubes rather than squares.

6. Meshing of the three dimensional was time taking and to avoid the time taking it was needed to adopt the brick element rather than tetrahedral elements.

7. Ux =0, degree of freedom was constrained than the displacement in the x-direction will taken as zero.

8. Uy=0, degree of freedom was constrained than the displacement in the x-direction will taken as zero.

9. All Degree of freedom were constrained and enter a displacement value of zero.

8. The refined mesh being generated adjacent to the vertical cut.

b) Approach of FEM 3Dimensional Vertical Excavation for single element

$$\{\varepsilon\} = \begin{bmatrix} & 010000000 & 00 & 0 \\ 0 & 000001 & 0000 & 0 \\ & 000000000 & 0 & 1 \end{bmatrix} \{\alpha\}$$

$$\begin{bmatrix} & 010000000 & 00 & 0 \\ 0 & 000001 & 0000 & 0 \\ & 000000000 & 0 & 1 \end{bmatrix} \{\alpha\}$$

$$\{\varepsilon\} = [B] \{\alpha\}$$

$$\{\varepsilon\} = [B] \left[A^{-1}\right] \{q\}$$

$$\{q\} = [A] \{\alpha\}$$

$$[A^{-1}] = \begin{bmatrix} \{A1^{-1}\} & \{0\} \\ \{0\} & \{A1^{-1}\} \end{bmatrix}$$

$$[A^{-1}] = \begin{bmatrix} [A1^{-1}] & 0 & 0 \\ 0 & [A\text{-}1^{1}] & 0 \\ 0 & 0 & [A1^{-1}] \end{bmatrix}$$

Approach of FEM 3Dimensional Vertical Excavation for simulation for Three dimensional excavation: **x-y plane**

$$\sum_{J=i}^{n} e\, i = \Sigma e\ddot{o} + \sum_{i=1}^{n} ei \qquad \text{...(i)}$$

y-z plane

$$\sum_{J=i}^{n} e\, i = \Sigma e\ddot{o} + \sum_{i=1}^{n} ei \qquad \text{... (ii)}$$

y-z plane

$$\sum_{J=i}^{n} e\, i = \Sigma e\ddot{o} + \sum_{i=1}^{n} ei \qquad \text{...(iii)}$$

Summarization in all three plane such as x-y-z were made, final stresses for each element with six shear stresses and three normal stressed were evaluated by means of FEM process .

c) Three Dimensional Displacement Models of ath orders

$u (x.y,z)= \alpha 1 + \alpha 2x + \alpha 3y + \alpha 4z + \alpha 5\ zx + \ldots\ldots\ldots + \alpha\ m\ z^{a}$

$v(x.y,z)= \alpha m{+}1 + \alpha m{+}2x + \alpha m{+}3y + \alpha m{+}4z + \alpha m{+}5\ zx + \ldots\ldots\ldots + \alpha 2m\ z^{a}$

$w(x.y,z)= \alpha 2m{+}1 + \alpha 2m{+}2\ x + \alpha 2m{+}3\ 2y + \alpha 2m{+}4\ z + \alpha 2m{+}5\ zx + \ldots + \alpha\ 3m\ z^{a}$

$$m = \sum_{i=1}^{a+1} 1(a+2+i)$$

u,v, and w were the components of displacement.

d) Bandwidth

The semi bendwidth B,

B = (D + 1) f

f = number of degree of freedom

e) Modified Vonmises Theory Equation:

(({61})–(62}) + 0.069({61}))² +(({62})– ({63}))² +(({63}) + ({61})1- .069({61}))² = Constant

f) Von Mises Theory and their correlations Equation:

(({61})1 –({62})2)² +(({262})2 – ({63}))² +(({63}) – ({61}))² = Constant

III. RESULT OF FEM: 3D-EXCAVATION

Nodes Number	Sxx*10e-6	Syy*10e-6	Sxy*10-6	Remark
1	-0.82756E-01	-0.81875E-01	-0.66186E-01	The Discretized continuum element number nodes are verified with the helpTriaxial Test . Stresses are in N/mm2
2	-0.55258E-03	-0.54910E-03	0.44290E-03	
3	-0.56757E-01	-0.54910E-03	0.44290E-03	
4	-0.56757E-01	-0.20949E-01	-0.10805E-01	
5	3.3940	4.2579	-12.765	
6	-1.3245	-1.2947	0.44061	
7	-0.11377E-01	-0.56088E-02	0.16207E-02	
8	12.080	-1.6516	-8.4834	
9	-1.4379	-12.065	-1.8639	
10	-6.0471	-11.625	-0.28453	
11	-5.9927	-9.4297	-0.24410E-01	
12	0.57527	1.1410	0.37600	
13	0.41445	0.81283	0.19396	
14	-0.27073E-01	0.19381	-0.76130E-03	

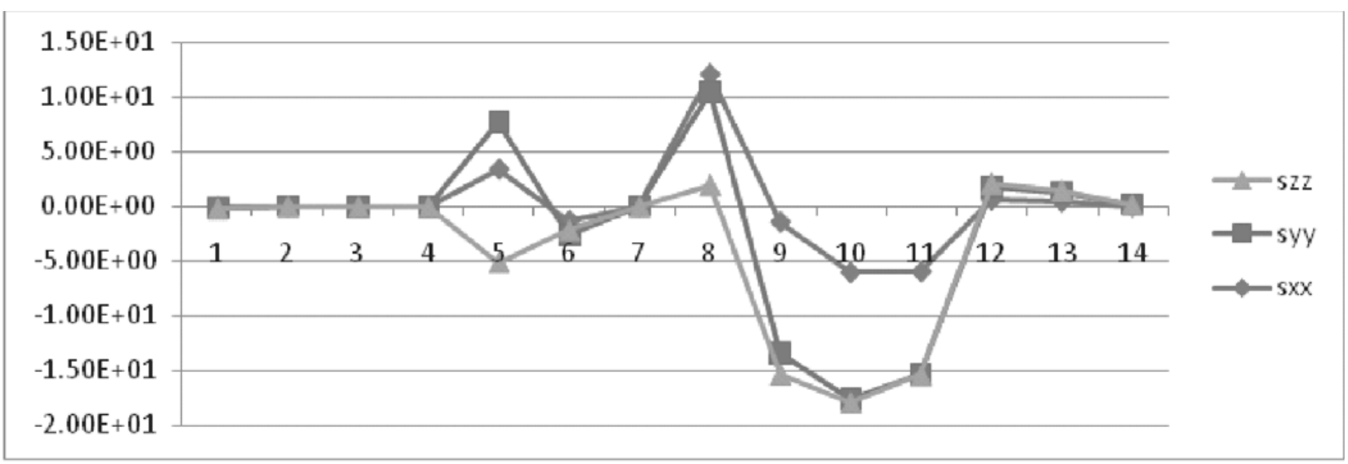


Fig. 1. Three Dimensions Stress Analysis: Nodal Number vs stresses

IV. EXPERIMENTAL SETUP

i) The Test equipment specially consists of a high pressure cylinder cell, made of Perspex or other transparent materials.fitted between the base and the top cap.

ii) Three out let connections are generally provided through the base: Two cell fluid inlet on either side, pore water out let from the bottom of the specimen and drainage out let from the top of the specimen. A separate compressor is used to apply fluid pressure in the cell.

iii) Pore pressure developed in the specimen during the test can be measured with the help of a separate pore pressure measuring equipment such as Bishop's apparatus. The cylindrical specimen is enclosed in the rubber membrane and the Triaxial Box is partitioned with the help of Steel sheets with an opening of specimen and the the fluid pressure is applied from the other side as per the obtained value of intermediate and minor stresses during the test procedures and the Exact major principle stresses deviatory stresses are determined at the failures of the specimen.

iv) The obtained exact value are used to obtained the shear stresses and the verification of the Shear stresses are obtained with the help of Mohr's circles. The % of the variation are limited to 5-6.4% in the case study of 14 nodes.

v) A Stainless steel piston running through the center of the top cap applies the vertical compressive load on the specimen under test. The load is applied thorough a proving ring with the help of a mechanically operated load frame.

vi) Depending upon the drainage condition of the test, solid non porous disc or end caps or porous discs are placed on the top and bottom of the specimen and the rubber membrane is sealed on to these end caps by rubber rings.

vii) The length of the specimen is kept about 2 to 21/2 times its diameter.

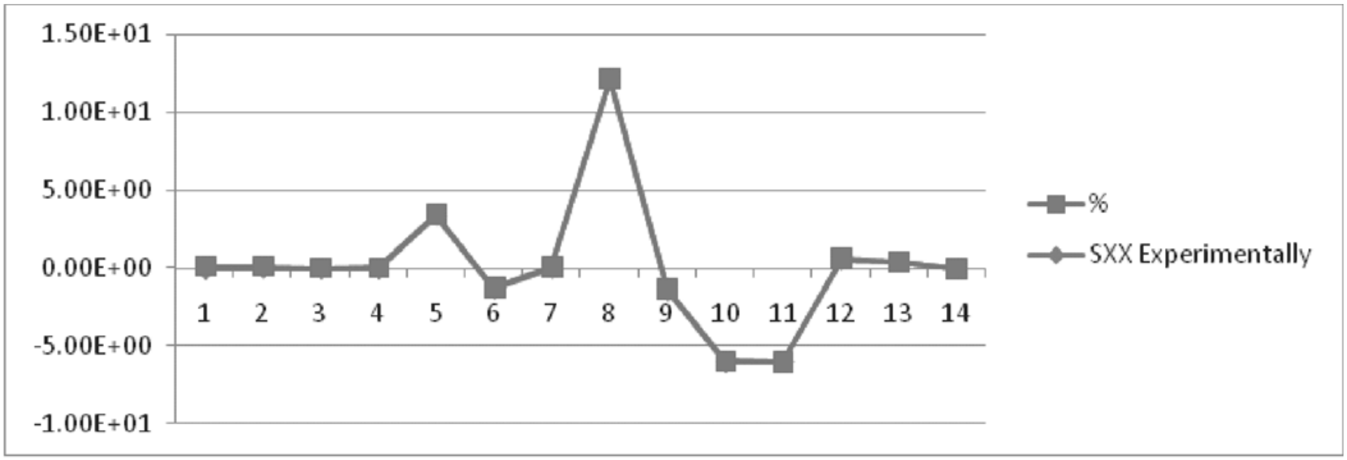
viii) The cell pressures minor principal stresses and intermediate acting on both side of specimen with the fluid, vertical piston means for application of the deviatory stresses on the top of the specimen.

ix) Mercury manometer is used for the measurement of negative pore pressures, accurate measurement of low positive pore pressure, zero errors of the pressure gauge. The valve are being closed which are not in use during the manometer observation.

V. EXPERIMENTAL RESULT VERIFICATION

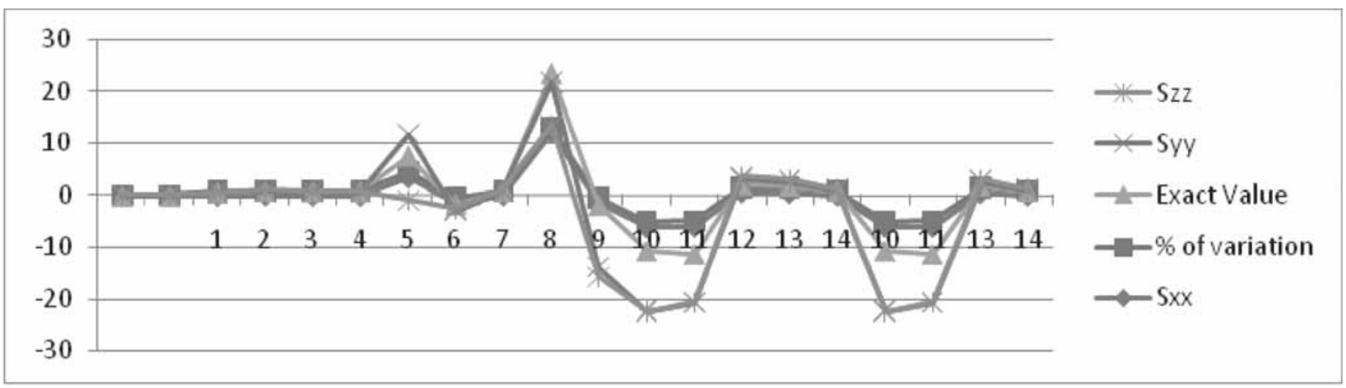
Nodes No.	Sxx*10e-6 N/mm2	Experimental Verification (% of verification Obtained applied Modified Triaxial Test)	Sxx*10e-6 N/mm2 Experimental verification	Syy* 10e-6 N/mm2	Experiment verification (same value applied Modified Triaxial Test)	Sxy* 10e-6 10e-6 N/mm2	Experimental Verification (same value applied Modified Triaxial Test)	Nodes No.
1	-0.82756E-01	20%	-6.62E-02	-0.81875E-01	-0.81875E-01	-0.66186E-01	-0.66186E-01	
2	-0.55258E-03	12%	-4.86E-04	-0.54910E-03	-0.54910E-03	0.44290E-03	0.44290E-03	
3	-0.56757E-01	6%	-5.34E-02	-0.54910E-03	-0.54910E-03	0.44290E-03	0.44290E-03	
4	-0.56757E-01	8%	-5.22E-02	-0.20949E-01	-0.20949E-01	-0.10805E-01	-0.10805E-01	
5	3.3940	4%	3.26E+00	4.2579	4.2579	-12.765	-12.765	
6	-1.3245	10%	-1.19E+00	-1.2947	-1.2947	0.44061	0.44061	
7	-0.11377E-01	11%	-1.01E-02	-0.56088E-02	-0.56088E-02	0.16207E-02	0.16207E-02	
8	12.080	12%	1.06E+01	-1.6516	-1.6516	-8.4834	-8.4834	
9	-1.4379	8%	-1.32E+00	-12.065	-12.065	-1.8639	-1.8639	
10	-6.0471	8%	-5.56E+00	-11.625	-11.625	-0.28453	-0.28453	
11	-5.9927	-6%	-6.35E+00	-9.4297	-9.4297	-0.24410E-01	-0.24410E-01	
12	0.57527	7%	5.35E-01	1.1410	1.1410	0.37600	0.37600	
13	0.41445	-5%	4.35E-01	0.81283	0.81283	0.19396	0.19396	
14	-0.27073E-01	2%	-2.65E-02	0.19381	0.19381	-0.76130E-03	-0.76130E-03	

a) **Y-Y , N/m2,x-x, Nodes Numbers, Experimental verification of Sxx with reference to %, Sxx % 1-2-3-4 & 6-7**

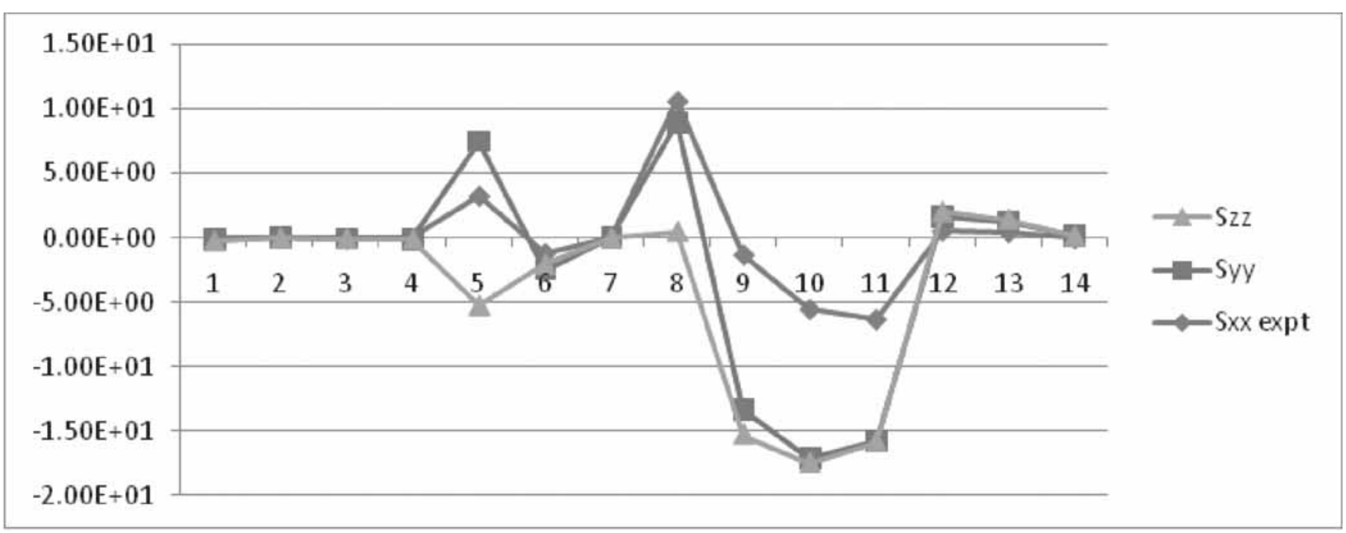


Nodal Number Vrs Stresses in N/m2

b) **Stresses N/m2 in Y-Y, Nodes numbers x-x, Three dimensional experimental verification chart, 1st lower Szz, 2nd lower Syy, 3rd lower Exact value**



c) **Sxx Experimental data, 3rd lowers, Szz FEM data, 1st lowers, Syy FEM data, 2nd lowers, Three dimensional Experimentally verified**



Number of nodes Vrs Stresses N/m2

VI. RESULT DISCUSSION AND CONCLUSION

Three dimensional stresses analysis and their experimental verification with triaxial undrained condition was established. The major stresses were as -0.82756e-7 N/mm2, whereas with the experimental verification the obtained value were 20% higher than what obtained with the help of traixial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM -0.818e-7 N/mm2 and -0.66186e-07 N/mm2 (For nodal point number 1).

i. The major stresses were as 0.55258E-09N/mm2, whereas with the experimental verification the obtained value were 12.0% higher than what obtained with the help of traixial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM 0.54910E-09N/mm2 and 0.44290E-09 N/mm2 (For nodal point number 2).

ii. The major stresses were as 0.56757E-07N/mm2, whereas with the experimental verification the obtained value were 12.0% higher than what obtained with the help of Triaxial test ,however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM 0.54910E-09N/mm2 and 0.44290E-09N/mm2 (For nodal point number 3).

iii. The major stresses were as 0.56757E-07N/mm2, whereas with the experimental verification the obtained value were 6.0% higher than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM 0.54910E-09N/mm2 and 0.44290E-09N/mm2 (For nodal point number 3).

iv. The major stresses were as -0.56757E-07 N/mm2, whereas with the experimental verification the obtained value were 6.0% higher than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM- 0.20949E-07N/mm2 and -0.10805E-07N/mm2 (For nodal point number 4).

v. The major stresses were as 3.3940e-6 N/mm2, whereas with the experimental verification the obtained value were 4.0% lower than what obtained with the help of triaxial test ,however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM, 4.2579e-6 N/mm2 and -12.765e-6 N/mm2 (For nodal point number 5).

vi. The major stresses were as-1.3245e-6 N/mm2, whereas with the experimental verification the obtained value were 10.0% higher than what obtained with the help of triaiaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM, -1.2947e-6 N/mm2 and 0.44061e-6 N/mm2 (For nodal point number 6).

vii. The major stresses were as 0.11377e-7 N/mm2, whereas with the experimental verification the obtained value were 11.0% higher than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM, 0.56088e-8 N/mm2 and 0.16207e-6 N/mm2 (For nodal point number 7).

viii. The major stresses were as12.080e-67 N/mm2, whereas with the experimental verification the obtained value were 11.0% higher than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the traiaxial box were as obtained with the FEM, -1.6516e-6 N/mm2 and -8.4834e-6N/mm2 (For nodal point number 8).

ix. The major stresses were -1.4379e-6N/mm2, whereas with the experimental verification the obtained value were 12.0% higher than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM -1.6516e-06N/mm2 and -8.4834e-6 N/mm2 (For nodal point number 9).

x. The major stresses were -6.0471e-6N/mm2, whereas with the experimental verification the obtained value were 8% higher than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM-11.625e-6N/mm2 and -0.28453e-6 N/mm2 (For nodal point number 10).

xi. The major stresses were -5.9927e-6N/mm2, whereas with the experimental Verification the obtained value were 6 % lower than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM-9.4297e-6N/mm2 and -0.24410E-07 N/mm2 (For nodal point number 11).

xii. The major stresses were -5.9927e-6N/mm2, whereas with the experimental verification the obtained value were 7% higher than what obtained with the help of triaxial test, however the other intermediate and minor stresses obtained were kept as same applied through the fluid pressure in the triaxial box were as obtained with the FEM1.1410e-6N/mm2 and 0.37600e-06 N/mm2 (For nodal point number 12).

xiii. Variation of deviatory stresses with the original value as such 6.9 % which was taken as positive, so the final stresses for the analysis will be additive of 6.9%. However the Deviatory stresses were obtained had a difference with the obtained value of FEM, Leads to a exact value. At the higher depth of excavation the boundary of the cut were affected and displacement vector were drop down showing the huge stresses were released and the exact analysis were possible.

xiv. Variation of deviatory stresses with the original value as such 6.9% which was taken as positive, so the final stresses for the analysis will be additive of 6.9%. However the Deviatory stresses were obtained had a difference with the obtained value of FEM, Leads to a exact value. At the higher depth of excavation the boundary of the cut were affected and displacement vector were drop down showing the huge stresses were released and the exact analysis were possible.

xv. The Wide variation of shear stresses were obtained at the depth of 6.0m and below the depth of 6.0m. The shear stresses contours shifting towards the bottom of boundary of vertical excavation.

xvi. The Tensile stresses were observed at the higher depth of the excavation and required extreme care during the excavation of vertical cut.

xvii. The variation of Deviatory stresses were experimentally verified found 7.5% Higher, however the Shear Stresses variation were obtained as 7.5% higher. Hence for deeper excavation the shear stresses contours showed the failure mechanism develop hence suitable measures were needed during the higher depth of excavation and the stresses were experimentally verified so the design value Fem may be taken as 7.5% higher than the value obtained with FEM.

xviii. Variation of deviatory stresses with the original value as such 6.9% which was taken as positive, so the final stresses for the analysis would be additive of 6.9%. However the Deviatory stresses were obtained had a difference with the obtained value of FEM, Leads to a exact value if the equation were modified.

xix. At the higher depth of excavation the boundary of the cut were affected and displacement vector were drop down showing the huge stresses were released and the exact analysis are possible.

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