Ant Colony Optimization: A Tutorial Review

Sapna Katiyar, Ibraheem
Department of Electrical Engineering
Jamia Millia Islamia, New De1lhi- 110 025
Email : sapna_katiyar@yahoo.com,
ibraheem_2k@yahoo.com

Abdul Quaiyum Ansari*
*Corresponding Author:
(Email: aqansari@ieee.org)

Abstract: *The complex social behaviors of ants have been much studied, and now scientists are finding that these behavior patterns can provide models for solving difficult combinatorial optimization problems. The attempt to develop algorithms inspired by one aspect of ant behavior, the ability to find shortest paths, has become the field of ant colony optimization (ACO). Ant Colony Optimization (ACO) is a derivative of Swarm intelligence (SI). The ant colony optimization algorithm (ACO), introduced by Marco Dorigo, in the year 1992 and it is a paradigm for designing meta heuristic algorithms for optimization problems and is inspired by the foraging behavior of ant colonies. Ant Colony Optimization targets discrete optimization problems and can be extended to continuous optimization problems which is useful to find approximate solutions. Now-a-days, a number of algorithms inspired by the foraging behavior of ant colonies have been applied to solve difficult discrete optimization problems. In fact, ACO algorithm is the most successful and widely recognized algorithm based on the ant behavior. This paper gives an overview of growing research field from theoretical inception to the practical applications of ACO variants and some of the fields where it can be applied.*

Keywords: *Hybridization, Metaheuristic, Parameters optimization, Pseudo-Random-Proportional Action Choice Rule, Pheromone*

I. INTRODUCTION

A. *Ant Behavior*

Ants communicate to one another by laying down pheromones along their trails, so where ants go within and around their ant colony is a stigmergic system. In many ant species, ants walking from or to a food source, deposit on the ground a substance called pheromone. Other ants are able to smell this pheromone, and its presence influences the choice of their path, that is, they tend to follow strong pheromone concentrations. The pheromone deposited on the ground forms a pheromone trail, which allows the ants to find good sources of food that have been previously identified by other ants. Ant behavior is shown in Fig. 1. Using random walks and pheromones within a ground containing one nest and one food source, the ants will leave the nest, find the food and come back to the nest. After some time, the way being used by the ants will converge to the shortest path.

- Ants in a pheromone trail between nest and food;
- An obstacle interrupts the trail;
- Ants find two paths to go around the obstacle;
- A new pheromone trail is formed along the shorter path.

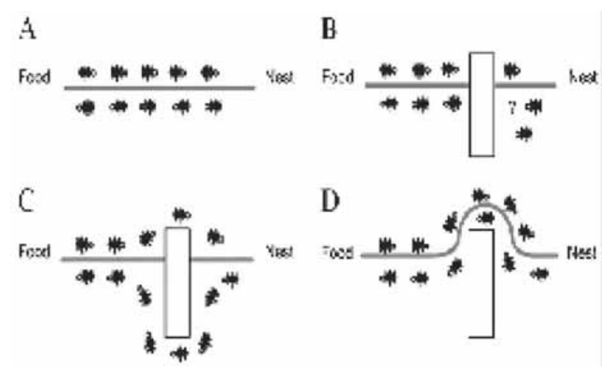

Fig. 1: Ant Behavior

B. *Ant colony optimization: A metaheuristic*

The term *metaheuristic* is a combination of two Greek words. *Heuristic* derives from the verb *heuriskein* which means “to find”, while the suffix *Meta* means “beyond, in an upper level”. The new heuristic has the following desirable characteristics:

- *Versatile:* It can be applied to similar versions of the same problem; for example, there is a straightforward extension from the traveling salesman problem (TSP) to the asymmetric traveling salesman problem (ATSP).

- *Robust*: It can be applied with only minimal changes to other combinatorial optimization problems such as the quadratic assignment problem (QAP) and the job-shop scheduling problem (JSP).

- *Population based approach:* This is interesting because it allows the exploitation of positive feedback as a search mechanism, as explained later in the paper. It also makes the system amenable to parallel implementations.

Ant system is the first member of ACO class of algorithms. This algorithm is inspired by the trail laying and following behavior of natural ants. The essential trait of ACO algorithm is the combination of priori information about the structure of a promising solution with posterior information about the structure of previously obtained good solutions. The main underlying idea, loosely inspired by the behavior of real ants, is that of a parallel search over several constructive computational threads based on local problem data and on a dynamic memory structure containing information on the quality of previously obtained result.

C. The Double Bridge Experiment

Deneubourg et al. verified the pheromones marking of ants by the experience known as "double bridge experiment". This configuration is as shown in Fig. 2. In the double bridge experiment, a nest of a colony of Argentine ants is connected to a food source by two bridges. The ants can reach the food source and return to the nest using any of the two bridges. The aim of the experiment is to observe the resulting behavior of the colony.

It is observed that if the two bridges have the same length, the ants tend to converge towards the use of one of the two bridges. If this experiment is repeated a number of times, it is observed that each of the two bridges is used in about half of the cases. The fact is while moving, ants deposit pheromone on the ground and whenever they have to decide which path should follow, their choice is based higher the pheromone concentration found on a particular path, the higher is the probability to follow that path.

II. ANT COLONY OPTIMIZATION ALGORITHM

When ants move from point A (source) to point B (destination), ants leave a chemical, pheromone, to mark these paths. This helps the following ants to find the way of their team members as they detect pheromone and choose, in probability, paths having greater concentration of pheromone. The algorithm is based on adaptively adjusting the pheromone on routes at each node and is shown in Fig. 3. Choice of this node is guided by a probability based selection approach.

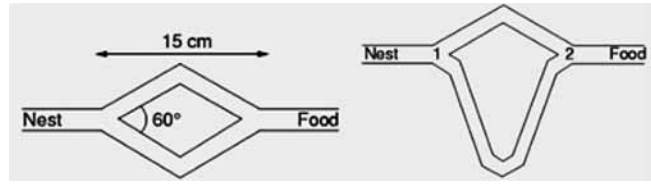

Fig. 2: Experimental Setup for Double Bridge Experiment: (a) Branches have equal length. (b) Branches have different length

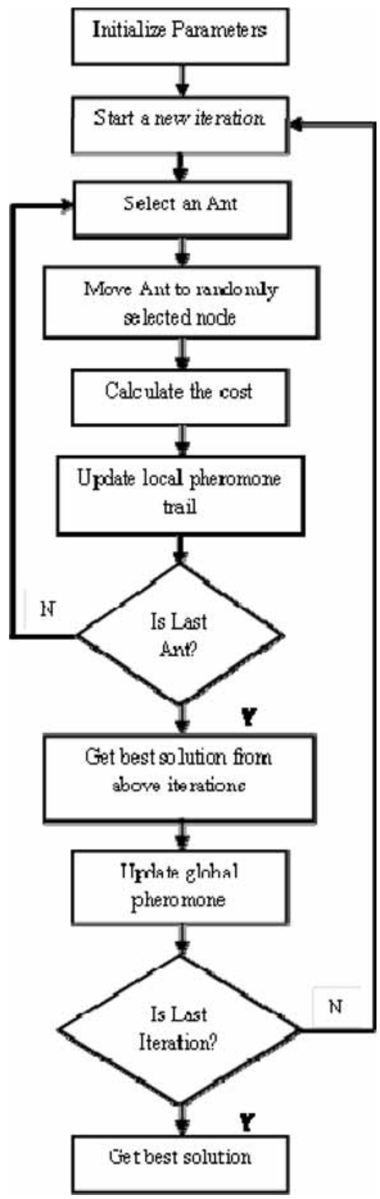

Fig. 3: ACO Algorithm

The ants are driven by a probability rule to choose their solution to the problem, known as a tour. The probability rule between two nodes j, called Pseudo-Random-Proportional Action Choice Rule, and it depends on two factors: the heuristic and metaheuristic.

$$p_{ij} = \frac{\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{h \in s}[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}} \qquad (2.1)$$

Where τ is the pheromone, η is the inverse of the distance between the two nodes. Each ant modifies the environment in two different ways:

i) *Local trail updating:* As the ant moves between nodes it updates the amount of pheromone on the edge by the following equation:

$$\tau_{ij}(t) = (1-\rho).\tau_{ij}(t-1) + \rho\,\tau \qquad (2.2)$$

Where, ñ is the evaporation constant. The value ô0 is the initial value of pheromone trails and can be calculated as:

$$\tau 0 = (n / L_n)\text{-}1$$

Where, n is the number of nodes and L_n the total distance covered between the total nodes, produced by one of the construction heuristics.

ii) *Global trail updating:* When all ants have completed all the nodes that find the shortest path updates the edges in its path using the following equation:

$$\tau_{ij}(t) = (1-\rho).\tau_{ij}(t-1) + \frac{\rho}{L^{+}} \qquad (2.3)$$

Where, L^{+} is the length of the best path generated by one of the ants.

III. VARIANTS OF ACO ALGORITHM

In literature, number of ACO algorithms have been proposed. The original ACO algorithm is known as Ant System and was proposed in the early nineties. Here we demonstrate the original Ant System (AS) and two other most successful variants: MAX-MIN Ant System and Ant Colony System (ACS).

A. *Ant System (AS)*

Ant System (AS) was the first (1991) ACO algorithm proposed in literature. Its importance is mainly because of the prototype of a number of ant algorithms which have found many interesting and successful applications in real time situations. Three AS algorithms have been defined and they differ by the way pheromone trails are updated. These algorithms are:

- Ant-density
- Ant-quantity
- Ant-cycle

In ant-density and ant-quantity ants deposit pheromone during building a solution, while in ant-cycle ants deposit pheromone after the completion of a complete tour.

$$\tau_{ij}(t) = (1-\rho).\tau_{ij}(t-1) + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \qquad (3.1)$$

Where ρ is the evaporation rate, m is the number of ants and $\Delta\tau^{k}_{ij}$ is the quantity of pheromone laid on edge (i, j) by ant k: τ_{ij} (t) = Q / L_k, if ant k used edge (i, j) in its tour otherwise it is zero. Where, Q is a constant and L_k is the length of the tour constructed by ant k.

B. *MAX–MIN Ant System*

MAX–MIN Ant System (MMAS) is another improvement over the original ant system and it is proposed by Stützle and Hoos (2000). It introduces four main modifications with respect to ant system.

- Only the best ant adds pheromone trails i.e. either the ant that produced the best tour in the current iteration, or the best-so-far ant is allowed to deposit pheromone. But this may lead to a stagnation situation.
- Above defined situation can be modified by MMAS i.e. the minimum and maximum values of the pheromone are explicitly limited
- The pheromone trails are initialized to the upper pheromone trail limit, which, together with a small pheromone evaporation rate which increases the exploration of tours at the start of the search.
- Finally, in MMAS, pheromone trails are reinitialized each time the system approaches stagnation or when no improved tour has been generated for a certain number of consecutive iterations.

When all ants have constructed a tour, pheromones are updated by applying evaporation same as in the case of ant system (AS), followed by the deposit of new pheromone given by:

$$\tau_{ij}(t) \leftarrow \tau_{ij} + \Delta\tau_{ij}^{best} \quad (3.2)$$

Where $\Delta\tau_{ij}^{best} = 1/C_{best}$, if the best ant used edge (i, j) in its tour. $\Delta\tau_{ij}^{best} = 0$ otherwise, where L_{best} is the length of the tour of the best ant. The value of L_{best} may be set either to L_{ib} or to L_{bs}, or to a combination of both.

C. Ant Colony System (ACS)

The difference between ACS and ant system is because of three main aspects:

- The state transition rule provides a direct way to balance between exploration of new edges and exploitation of a priori and accumulated knowledge about the problem.
- The global updating rule is applied only to edges which belong to the best ant tour
- While ants construct a solution a local pheromone updating rule is applied.

The most interesting contribution of ACS is the introduction of a local pheromone update in addition to the pheromone update performed at the end of the construction process (called offline pheromone update). The local pheromone update is performed by all the ants after each construction step.

IV. APPLICATIONS OF ANT COLONY OPTIMIZATION

ACO was first introduced for the application of travelling salesman problem (TSP). Since then, it has been applied to many discrete optimization problems. Initially it is applied to classical problems like assignment problems, graph coloring, the maximum clique problem, scheduling problems and vehicle routing problems. Some of the examples of more recent applications include the intelligent scheduling, design of communication networks, bioinformatics problems, cell placement problems arising in circuit design and machine learning. In recent years some researchers have also focused on the application of ACO algorithms to multi-objective problems and to dynamic or stochastic problems. Table 1 shows some successful applications of ACO.

Table 1: A non-exhaustive list of successful ant colony optimization algorithms

S. No	Problem type	Problem name
1	Routing	Traveling salesman (TSP)
		Vehicle routing
		Sequential ordering
		Connection-oriented network routing
		Connectionless network routing
		Optical network routing
2	Assignment	Quadratic assignment
		Course timetabling
		Graph coloring
		Frequency Assignment
3	Scheduling	Project scheduling
		Car sequencing
4	Subset	Set covering
		Multiple knapsack
		Maximum clique
5	Machine learning	Classification rules
		Bayesian networks
		Neural networks
		Fuzzy Systems
6	Bioinformatics	Protein folding
		DNA Sequencing

A. Travelling Salesman Problem (TSP)

In the traveling salesman problem, a set of cities is given and the distance between each of them is known. The goal is to find the shortest tour that allows each city to be visited once and only once. In more formal terms, the goal is to find a Hamiltonian tour of minimal length on a fully connected graph.

In ant colony optimization, the problem is tackled by simulating a number of artificial ants moving on a graph that encodes the problem itself: each vertex represents a city and each edge represents a connection between two cities. A variable called pheromone is associated with each edge and can be read and modified by ants. The traveling salesman problem plays a central role in ant colony optimization because it was the first problem to be attacked by ACO.

The TSP was chosen because:

- It is relatively easy to adapt the ant colony metaphor to it.
- It is a very difficult problem (NP-hard).
- It is one of the most studied problems in combinatorial optimization.

· It is very easy to state and explain, so that the algorithm behavior is not obscured by too many technicalities.

B. Quadratic Assignment Problem (QAP)

The quadratic assignment problems (QAPs) are the problem of assigning 'n' facilities to 'n' locations so that the assignment cost is minimized, where the cost is defined by a quadratic function. A concrete example would be a hospital planning optimization the location of various medical facilities in order to minimize the overall travel distance of patients inside the hospital.

C. Vehicle Routing Problem (VRP)

A direct extension of the TSP is Vehicle routing problem (VRP). These are problems where a set of vehicles stationed at a depot has to serve a set of customers before returning to the depot, and the objective is to minimize the number of vehicles used and the total distance traveled by the vehicles. Capacity constraints are imposed on vehicle trips, as well as possibly a number of other constraints deriving from real-world applications, such as time windows, backhauling, rear loading, vehicle objections, maximum tour length, etc.

D. Graph Coloring Problem (GCP)

One of the most well-studied combinatorial optimization problems is the graph coloring problem. The problem is to find a coloring of the vertices with minimum number of colors such that no pair of adjacent vertices has the same color. Graph coloring problem is expected to have a wide variety of applications such as:

- Scheduling,
- Frequency assignment in cellular networks,
- Timetabling,
- Crew assignment

E. Sequential Ordering Problem (SOP)

The sequential ordering problem is closely related to the asymmetric TSP, but additional precedence constraints between the nodes have to be satisfied. Gambardella and Dorigo have tackled this problem by an extension of ACS enhanced by a local search algorithm (Gambardella & Dorigo, 1997). They obtained excellent results and were able to improve the best known solutions for many benchmark instances.

F. Job Scheduling Problem (JSP)

The job-shop scheduling problem is concerned with allocating limited resources to operations over time. Although the job shop scheduling has an important role in various fields, it is one of the most difficult problems in combinational optimization. The demand for scheduling is to achieve high performance computing. Typically, it is difficult to find an optimal resource allocation for specific job that minimizes the schedule length of jobs. The scheduling problem is defined NP-hard problem and it is not trivial.

G. Routing in Telecommunication Network

ACO algorithms have shown to be a very effective approach for routing problems in telecommunication networks where the properties of the system, such as the cost of using links or the availability of nodes, varies over time. ACO algorithms were first applied to routing problems in circuit switched networks (e.g. telephone networks) and then in packet-switched networks (e.g. LAN or the Internet). A well-known example for network communication improved to the point of being state-of-the-art in wired networks is Ant-Net. Ant-Net has been extensively tested, in simulation, on different networks and under different traffic patterns, proving to be highly adaptive and robust.

V. ADVANTAGES AND DISADVANTAGES OF ACO

There are several advantages and disadvantages of ant colony optimization. Some of them are listed in Table 2:

Table 2: Advantages & Disadvantages of ACO

S. No	Advantages	Disadvantages
1	Inherent parallelism	Theoretical analysis is difficult
2	There is no central control in the colony	Sequences of random decisions (not independent)
3	Positive Feedback accounts for rapid discovery of good solutions	Probability distribution changes by iteration
4	Efficient for Traveling Salesman Problem and similar problems	Time to convergence uncertain (but convergence is guaranteed)
5	Can be used in dynamic applications (adaptsto changes such as new distances, etc)	Research is experimental rather than theoretical

VI. CURRENT TRENDS IN ACO

It is a true metaheuristic, with dozens of application areas. While both the performance of ACO algorithms and our theoretical understanding of their working have significantly increased, as discussed earlier, there are several areas in which until now only preliminary steps have been taken and where much more research will have to be done.

One of these research areas is the extension of ACO algorithms to more complex optimization problems that include:

- Dynamic problems: in which the instance data, such as objective function values, decision parameters, or constraints, may change while solving the problem.
- Stochastic problems: in which one has only probabilistic information about objective function value(s), decision variable values, or constraint boundaries, due to uncertainty, noise, approximation, or other factors.
- Multiple objective problems: in which a multiple objective function evaluates competing criteria of solution quality.

Active research directions in ACO include also the effective parallelization of ACO algorithms and, on a more theoretical level, the understanding and characterization of the behavior of ACO algorithms while applying it to any proposed system.

VII. CONCLUSIONS

Ant Colony Optimization has been and continues to be a fruitful paradigm for designing effective combinatorial optimization solution algorithms. In this work we first gave a detailed description of the origins and the basics of ACO algorithms. Then we outlined the general framework of the ACO metaheuristic and presented some of the most successful ACO variants today. Generally majority of problems based on ACO are static and well defined combinatorial optimization problems where all necessary information related to the problem is available and does not change during problem solution. It has been also identified that ACO algorithms can be applied in a systematic manner to ill-structured problems for highly dynamic domains with only local information available or to such type of problems where it is not clear how to apply local search. After listing some applications area of ACO, it has been found that the field of ant colony optimization algorithms is very dynamic in both theoretically and experimentally. Experimentally the direction of current research is to increase the number of successfully solved problems based on ACO algorithm. It can be implemented in real-word industrial applications also.

Finally, a survey on very interesting recent trends in ACO is discussed. The hybridization of ACO algorithms with more classical artificial intelligence and operations research methods can be done. In the opinion of the author, this research direction offers many possibilities for valuable future research.

REFERENCES

[1] M. Dorigo. *Optimization, Learning and Natural Algorithms*, Ph.D. thesis, Dipartimento di Elettronica, Politecnico di Milano, Italy, 1992.

[2] M. Dorigo, & L. M. Gambardella, “Ant Colony System: A Cooperative Learning Approach to the Traveling Salesman Problem,” *IEEE Transactions on Evolutionary Computation,* 1(1), 53-66, 1997.

[3] M. Dorigo, V. Maniezzo, and A. Colorni, “Ant System: Optimization by a colony of cooperating agents,” *IEEE Transactions on Systems, Man, and Cybernetics – Part B*, vol. 26, no. 1, pp. 29–41, 1996.

[4] M. Dorigo and G. Di Caro, *“The Ant Colony Optimization Meta-heuristic”*, in *New Ideas in Optimization*, D. Corne et al., Eds., McGraw Hill, London, UK, pp. 11-32, 1999.

[5] Maniezzo V, Colorni A. “The Ant System applied to the quadratic assignment problem”, *IEEE Trans. Data Knowledge Engineering*, 11(5), 769–78, 1999.

[6] M. Dorigo, G. Di Caro, and L. M. Gambardella, “Ant algorithms for discrete optimization,” *Artificial Life*, Vol. 5, No. 2, pp. 137–172, 1999.

[7] M. Dorigo and T. St¨utzle, *Ant Colony Optimization*, MIT Press, Cambridge, MA, 2004.

[8] A. Q. Ansari, “The Basics of Fuzzy logic: A Tutorial Review,” Computer Education – Stafford – Computer Education Group, U.K., No. 88, pp. 5 - 9, February 1998.

[9] A. Q. Ansari, “Multiple Valued Logic Versus Binary Logic,” CSI Communications, India, Vol. 20, No. 5, pp. 30 - 31, November 1996.

[10] A. Q. Ansari, Moinuddin, S. Deshpande, “Fuzzy Logic Control of Safety and Security Systems,” The Indian Police Journal, Vol. 45, No. 4, pp.56 – 62, October–December 1998.

[11] S. A. Siddiqui, A. Q Ansari, Shikha Agarwal, “A Journey through Fuzzy Philosophy,” Pranjana, India, Vol. 6, No. 2, pp. 29 – 33, July – Dec. 2003.

[12] V. Kumar, A. Q. Ansari, T. Patki, A.B. Patki, S. Joshi, S. Chowdhary, “Applications of Evidence Based Software

Engineering for I T Systems," Review of Business and Technology Research, Vol. 1, No. 1, pp. 1 – 7, 2008.

[13] Abdul Quaiyum Ansari, *Hierarchical Fuzzy Control for Industrial Automation*, Scholar's Press., Germany, 07/2013; ISBN: 978-3639515923.

[14] Mohammad Ayoub Khan, Abdul Quaiyum Ansari, *Handbook of Research on Industrial Informatics and Manufacturing Intelligence: Innovations and Solutions*, IGI Global, USA, 2012; ISBN13: 9781466602946, ISBN10: 1466602945, EISBN13: 9781466602953.

[15] Abdul Quaiyum Ansari, Mohammad Ayoub Khan, "Fundamentals of Industrial Informatics and Communication Technologies," in *Handbook of Research on Industrial Informatics and Manufacturing Intelligence: Innovations and Solutions*, chapter 1, pages 1-19; IGI Global, USA, 2012; ISBN13: 9781466602946, ISBN10: 1466602945.

[16] Sapna Katiyar, Avneesh Mittal, A. Q. Ansari, T. K. Saxena, "Ant Colony Algorithm Based Adaptive PID Temperature Controller," *Proc. 7th Int. Conf. on Trends in Industrial Measurements and Automation* (TIMA 2011), CSIR, Chennai, January 2011.

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