Applicability of Fuzzy Logic in Clustering

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Abstract: *Fuzzy logic is used to make decisions better that suit the real world problem more perfectly. In traditional logic, decision are based on exact true or false values, which is sometimes inadequate, and lacks the flexibility in decision making, whereas, fuzzy logic provides a better approach in problem solving by evaluating many alternatives through simple If-Then statements, which resembles human reasoning greatly in handling of imperfect information. In this paper, we present the meaning of fuzzy logic, its applications and different aspects of fuzzy logic. Finally, utility and applicability of fuzzy logic in clustering has been proposed that may help in mining of data in data warehousing.*

1. Introduction

The concept of Fuzzy Logic was conceived by Lotfi Zadeh, a professor at the University of California at Berkley. In 1965, Zadeh presented "Fuzzy Sets" which described the mathematics of fuzzy set theory, and by extension fuzzy logic. A paradigm is a set of rules and regulations which defines boundaries and can accommodate the ambiguities of real-world problems to generate precise solutions from certain or approximate information within these boundaries. Fuzzy logic is a superset of conventional Boolean logic that has been extended to handle the concept of partial truth-false values between "completely true" and "completely false". It is the logic underlying modes of reasoning which are approximate rather than exact, thus resembles human reasoning and decision making. Fuzzy logic has proven to be an excellent choice for many control system applications since it mimics human control logic.

2. Overview of Fuzzy Logic

Fuzzy Logic, a subtopic of reasoning [1] and an extension of fuzzy set theory is basically a multi-valued logic [1] that allows a continuous range of intermediate truth values in the interval [0, 1] rather than the conventional strict binary evaluations like True/False, Yes/No, Black/White, etc [2]. Unlike classical logic (Boolean/crisp set theory), which requires a deep understanding and complete information of a system, exact equations, and precise numeric values, fuzzy logic incorporates an alternative way of thinking, which allows describing and modeling complex systems using a higher level of abstraction originating from our knowledge and experience in simple English-like rules. It provides a way of processing the data, representing subjective or vague human ideas such as "An attractive person" or "Pretty cold" to be formulated mathematically and can be processed by computers using simple rule-based (IF-AND-THEN) approach. Fuzzy logic fills an important gap in engineering design (system design) methods left vacant by purely mathematical approaches (e.g. linear control design), and purely logic-based approaches (e.g. expert systems) [3]. It provides a simple way to arrive at a definite, highly adaptive and a better conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information [4] with wide range of variations. The process of deriving an output from an input in fuzzy logic, is called "inference," is not a calculation; rather, we convert real-world inputs to fuzzy values, identify the applicable rules and derive a "weighted average i. e. degree of participation called possibility," and translate this information back to a real-world ("crisp," the complement of fuzzy) value. In technological terms, the process of deriving outputs from a series of input values with fuzzy logic involves "fuzzification," followed by inference, followed by "defuzzification." The input variables are "Fuzzy" or "linguistic" [23],[24] i.e. instead of using numbers we can think the certain variables in terms of objects or words and can phrase them as noun e.g. "temperature", "flow", "pressure" etc.

The essential characteristics of fuzzy logic may be as follows:

- In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.
- In fuzzy logic everything is a matter of degree.
- The boundaries of fuzzy sets are not rigid but fuzzy.
- Any logical system can be fuzzified.
- In fuzzy logic, knowledge is interpreted as a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.

- Inference is based on fuzzy-rules by calculating the rule's conclusion based on degree of participation.

3. Applications of Fuzzy Logic

Fuzzy logic can be built into anything from small, hand-held products to large computerized process control systems. It can be implemented in hardware, software, or a combination of both [4]. Fuzzy Logic has emerged as a profitable tool for the controlling and steering of systems and complex industrial processes, as well as for household and entertainment electronics, as well as for other expert systems [2] and applications like the classification of data in data mining, a technique of data warehousing. The main advantage of fuzzy logic is that it can be tuned and adapted if necessary, thus enhancing the degree of freedom of control [5]. Applications of fuzzy logic are as follows: -

- Control Systems (Robotics, Manufacturing and Process Control, Automation, Tracking, Consumer Electronics e.g. fuzzy washing machine, fuzzy microwave, and fuzzy air cooler).
- Information Systems (DBMS, Info. Retrieval).
- Pattern Recognition (Image Processing, Machine Vision).
- Decision Support (Adaptive HMI, Sensor Fusion).
- Medical imaging
- Financial trading

Before applying the fuzzy logic in clustering algorithm, let's briefly explain the important concepts like; fuzzy sets, membership functions and its degree of participation, fuzzy rules etc.

4. Fuzzy Set

Fuzzy set is a set with a smooth boundary. It generalizes the notion of membership from a black-and-white binary categorization in classical mathematics (crisp set theory) into one that allows the partial membership [6]. Let U is the universe of disclosure [17] such that U = {1, 2, 3...., 90}. A is a crisp sub-set from U where A= [10, 12]. Mathematically A= $\{x \mid 10 \le x \le 12, x \in U\}$. The value of x can only be 10, 12 or between 10 and 12. If any element is in A, it is assigned a degree 1 otherwise it is assigned 0, but can not be partial (between 0 and 1). Crisp set and fuzzy set are shown in below Figs. 1 & 2:

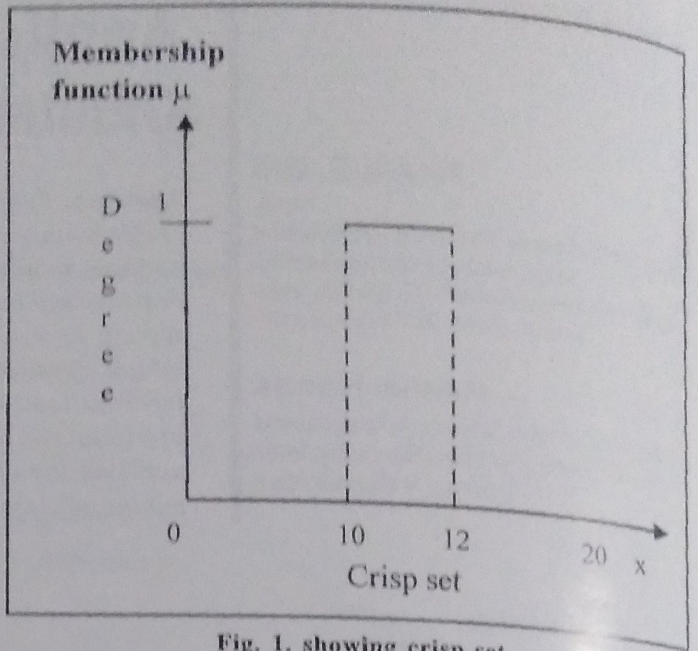

Fig. 1. showing crisp set

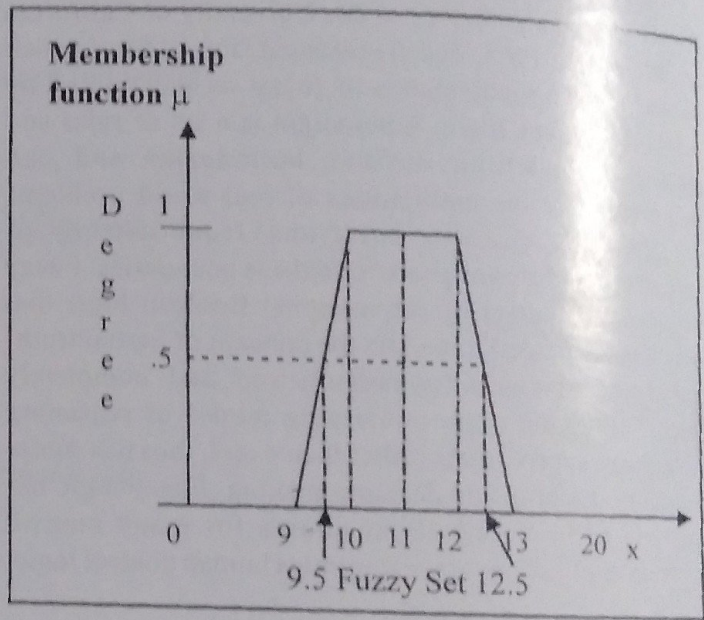

Fig. 2. showing fuzzy set

This way, crisp sets [17] lacks in flexibility for some applications, like classification of remotely sensed data analysis [2]. Fuzzy set is characterized by mapping from its universe of disclosure into intervals [0, 1] where a membership degree of 0 represents complete non-membership while a member degree of 1 represents a complete membership; variables having the degree in between represent partial membership in a fuzzy set which allowed many alternatives between 0 and 1. For example in temperature mapping, rather than putting a rigid boundary that we do in conventional sets, here we create fuzzy sets for temperature which suit the real life situation more perfectly, as shown by the following Figs. 3 & 4.

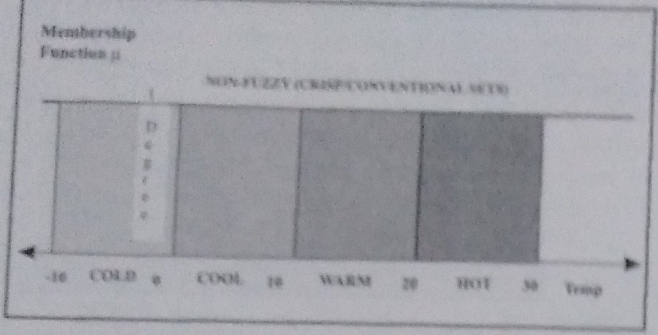

Fig. 3. Non-fuzzy sets for temperature.

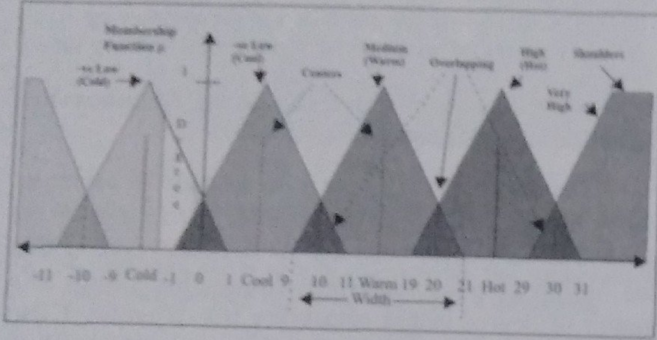

Fig. 4. Fuzzy sets to characterize the temperature using membership function, with overlapped area.

The above Fig. 4 showing the fuzzy sets for different temperatures, where everything is a matter of degree of participation in a fuzzy set, defined by its membership function μ. $\mu_{(Fuzzy\text{-}set)}$ *(input variable)* is the Degree i.e. 0&1 and in-between. HEIGHT or magnitude of a fuzzy set is the highest membership value of its membership function μ [6] and is usually normalized to 1 [4]: HEIGHT (Fuzzy set) = $max\ (\mu_{(Fuzzy\ set)}(x))$. WIDTH (is the base of function; space between lowest and highest value of any set on x-axis). SHOULDERING (locks height at maximum (1) if an outer function. Shouldered functions evaluate as 1.0 past their center [4]). CENTER points (center of the member function shape), OVERLAP (small triangles in fig.4, typically about- 50% of width: is maximum overlapping, but can be less). A fuzzy set with HEIGHT 1 is called a *normal fuzzy set* and a fuzzy set whose HEIGHT < 1 is called a *subnormal fuzzy set* [6]. Thus a *subnormal fuzzy set* introduces the area somewhere between two extremes: empty sets (a set with no members i.e. degree 0) and nonempty sets (a set with full members [17] i.e. degree 1), means containing only partial members. In the above Fig. 4, four fuzzy sets are there: Cold, Cool, Warm & Hot defined as follows:

Cold = $\{x \mid -11 \le x \le 1, x \in U\}$;
Cool = $\{x \mid -1 \le x < 11, x \in U\}$;
Warm = $\{x \mid 9 \le x \le 21, x \in U\}$;
Hot = $\{x \mid 19 \le x \le 31, x \in U\}$;
where U = $\{-11, \ldots, 31\}$;

We can perform the fundamental operations of classical sets: union, intersection and complement on fuzzy sets also [24]:

Cold $\cup$ Cool = $\{x \mid -11 \le x \le 11, x \in U\}$;
Cold $\cap$ Cool = $\{x \mid -1 \le x \le 1, x \in U\}$;
Warm $\cup$ Hot = $\{x \mid 9 \le x \le 31, x \in U\}$;
Warm $\cap$ Hot = $\{x \mid 19 \le x \le 21, x \in U\}$;
$Cold^c$ = $\{x \mid 1 < x \le 31, x \in U\}$;
$Cool^c$ = $\{x \mid -11 \le x < -1\ or\ 11 < x \le 31, x \in U\}$;
$Warm^c$ = $\{x \mid -11 \le x < 9\ or\ 21 < x \le 31, x \in U\}$;
Hot^c = $\{x \mid -11 \le x < 19, x \in U\}$.

5. Membership Function

The membership function is a graphical representation of the magnitude of participation of each input [4]. We have to assign a degree of membership [0 to 1] in fuzzy set, e.g. we called a person is Young depends upon its degree of participation in fuzzy set, or to decide whether a person is rich or not, we check if his annual income is High, where High is a fuzzy set defined by the membership function (not a crisp truth statement e.g. income>120k) shown by the Figs. 5 and 6:

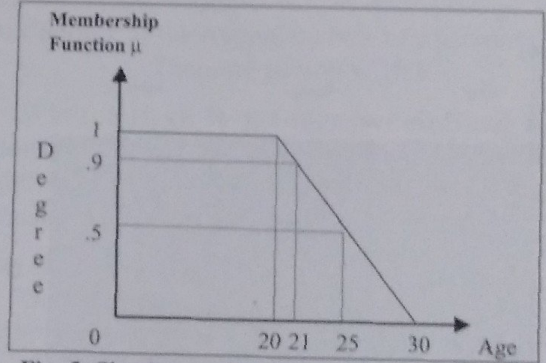

Fig. 5. Simple membership function & degree of participation in a fuzzy set

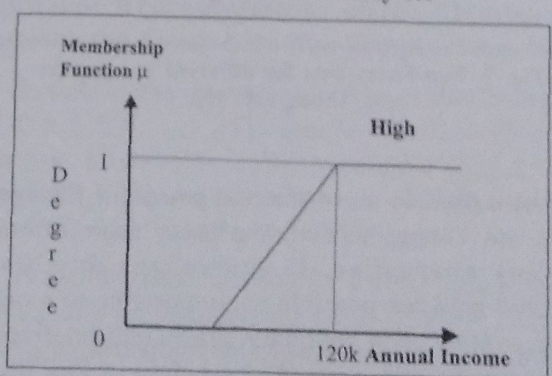

Fig. 6. Membership Function of High Annual Income

In the designing of membership functions, we ommonly used parameterized functions that can be efined by small number of parameters. The membership unction may be simple, based on single criteria/single arameter, for example: "Age" (called one-dimensional nembership functions); or may be more complex, based n two or more criteria/more than one parameter, for xample: "Age" & "Height" (called two-dimensional nembership functions). Using parameterized functions annot only reduce the system design time, it can also acilitate the automated tuning of the system because he desired changes in the membership function (e.g., videning vs. narrowing a membership function) can be lirectly related to corresponding changes in the related parameters [6]. The parameterized membership functions most commonly used are the triangular and rapezoid in shape but may be trapezoid, bell, and exponential too [4]. Numbers of parameters in triangular membership functions are three i.e. center, min and max values (Fig. 4), trapezoid membership functions has four parameters (Fig. 2). To know how the membership functions of fuzzy logic are more useful than the membership functions of traditional Boolean logic, we can do it by comparing them with the help of following example: in the below Fig. 7, there are three crisp/classical sets- Child, Young and Elder. The boundary for the set Young is set between 20 & 30, i.e. Age(x) = [20, 30] states that age can be 20, 21 ...29 or 30. Any other age: 19, 31 can not be included in set [20, 30].

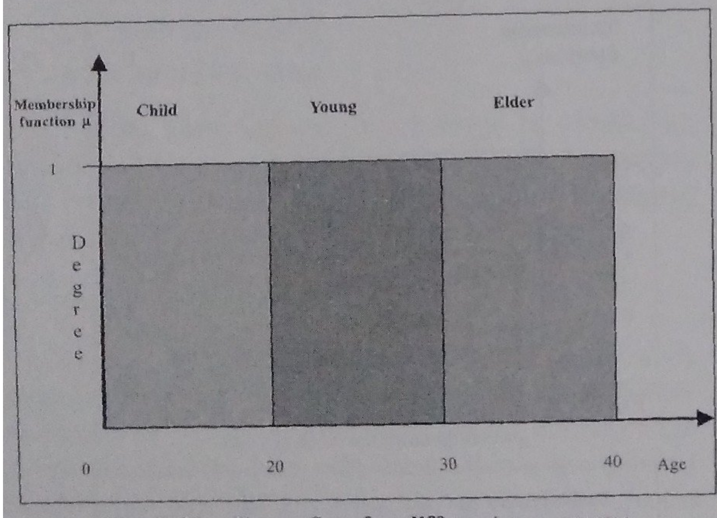

Fig. 7. Non-Fuzzy Sets for different age groups e.g. Young [20, 30].

But it doesn't mean that the person of the age 19 or 31is not Young, so here the fuzzy logic offers an appealing alternative. It generalizes this binary distinction between possible vs. impossible to a matter of degree (not a crisp boundary: 20-30 only) defined by membership functions, as shown below in Fig. 8.

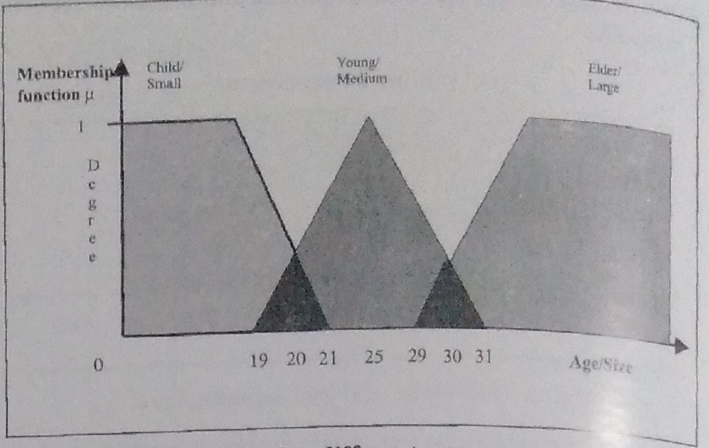

Fig. 8. Fuzzy Set for different age groups/siz... young [20, 30] using degree of membership in ... set.

Degree of Membership Defined by Me...rship Function of a Fuzzy Set:

Assigning a fuzzy set to a linguisti... iable, constrains the value of the variable, just as ... sp set does. The difference between the two, howe... is that the notion of possible versus impossible ...es is a matter of degree called possibility [6]. Whe...s, Crisp sets (using traditional logic) just set a rigid boundary for the degree of membership of each value (variable) in each set: if a variable is member, set it to 1 otherwise 0 (but between possible and impossible values, which may be undesirable in some situations); in case of fuzzy sets, we take the fuzzy variables: these may be the member of other Fuzzy sets also. For example: there are three sets for a given temperature: Low, Medium and High. In case of membership for the traditional sets (Fig. 9 below):- Low: only member variables of this Set are set to a degree 1 (from 0 to 10, on x-axis) and the degree of other variables (from the set Medium and set High) is set to 0. Likewise for the set- Medium: the degree of variables from 10 to 20 is set to 1, other variables from High and Low are set to a degree 0 for this set. For the set- High: only the variables from 20 to 30 are set to 1 and all other variables are set to 0 for this set as shown by the Fig. 9 below:

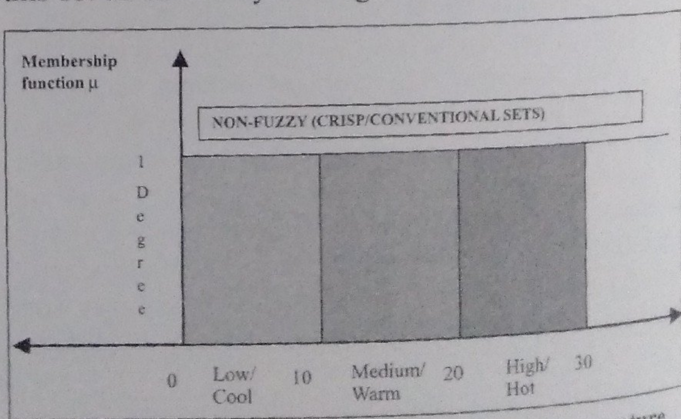

Fig. 9. Three Crisp sets (non-fuzzy sets) for temperature

In case of Fuzzy sets, the membership depends upon the degree rather than the rigid boundary. For example in the below Fig. 10 (with 50% overlapping): for the set Low, the degree of 9 is .55, degree of 10 is .5, degree of 17 is 0 where for the set Medium, degree of 9 is .4, degree of 10 is .5, degree of 17 is .7 and in set High, degree of 9 is 0, degree of 10 is 0, degree of 17 is .3 approximately as shown in Fig.10 below:

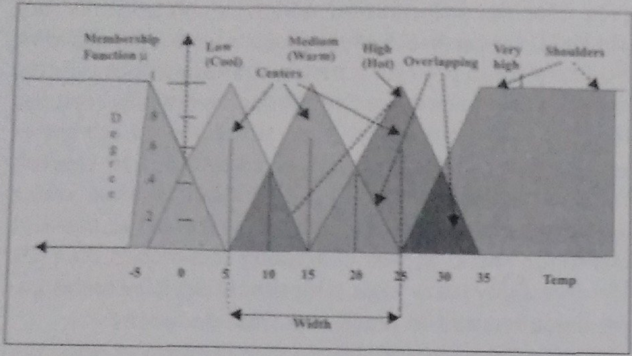

Fig. 10. Different Fuzzy sets to characterize the temperature using membership function, with 50% overlapping

One important thing about the membership function is that it provides a gradual transition from regions completely outside a set to regions completely in the set that is often exhibited in human reasoning [6]. It associates a weighting with each of the inputs that are processed, define functional overlap between inputs, and ultimately determines an output response [4]. We can compare the boundaries of the given sets: Low, Medium and High for traditional and fuzzy sets, in Table 1 as follows:

Table 1: Possible boundaries and center of three sets: Low, medium and High in fuzzy and crisp sets

Crisp Set Boundary	Fuzzy Set Boundary (with 50% overlapping)	Center
0 - 10	- 5 - 15	5
10 - 20	5 - 25	15
20 - 30	15 - 35	25

6. Fuzzy Rules in Fuzzy Logic

Human beings make the decisions based on some rules. These rules are computer-like If-Then statements, for example: If weather is fine... Then go out. Rules associates ideas and relate to one another [7]. Fuzzy logic enables the development of rule-based behavior i.e. the knowledge of an expert can be coded in the form of a rule-base, and used in decision making [5]. Most control applications have multiple inputs and require modeling and tuning of a large number of parameters which makes implementation very tedious and time consuming. Fuzzy rules can help you simplify implementation by combining multiple inputs into single if-then statements while still handling non-linearity [8]. These fuzzy rules operate using a series of If-Then statements [28] as humans, where the decisions are replaced by fuzzy sets and inputs are exchanged by fuzzy variables. From logical point of view, fuzzy rule-based inference is generalization of a logical reasoning scheme. Fuzzy rule is the basic unit for capturing knowledge in many fuzzy systems [6]. A fuzzy rule has two components: an If-part referred to as antecedent block (between If and Then) and a Then-part referred to as consequent block (following Then). The antecedent describes a condition and the consequent describes a conclusion that can be drawn when the condition holds.

Fuzzy rule-based inference

The algorithm of fuzzy rule-based inference may consist of following steps:

1. Fuzzy matching: Calculate the degree to which the input data match the condition of the fuzzy rules.
2. Inference: calculate the rule's conclusion based on its matching degree.
3. Combination: combine the conclusion inferred by all fuzzy rules into a final conclusion. For example:

Fuzzy Rule 1: If the Annual Income of a person is High, Then the person is *rich.*

Where: High is a fuzzy set defined by the membership function based on some degree for the condition of the rule, *rich* is the conclusion based on the matching degree.

Fuzzy Rule 2: If T is Low.
Then temperature is *cold.*
Where T is input variable for the temperature and *cold* is the decision based on matching degree.

Functional Representations of the Memberships Functions for Fuzzy Sets (for Temperature, Fig.10)

Lets U is the discrete universe of disclosure such that U= {-5,0, ..., 35}, where -5, 0,...,35 are its elements. Low= {-5,...15), Medium= {5,...25} and High= (15,...35} are fuzzy subsets of U characterized by the following membership functions:

A membership function for the set Low is defined by μ_{Low} and its membership value (degree) is defined by μ_{Low} (T), membership function for the set Medium is defined by μ_{Medium} and its membership value (degree) is defined by μ_{Medium} (T), membership function for the set High is defined by μ_{High} and its membership value is defined by μ_{High} (T); where T is an input variable defined for temperature.

Assumptions:

1. T is the variable defined for temperature (input).
2. *low* and *high* are the variables defined for lower and upper bounds (values on x-axis) of different fuzzy sets.
3. *cen* is a variable defined for the 'center' of different fuzzy sets; where:

   $cen = \frac{width}{2}$; and width = *low* + *high*,
4. $\mu_{(\text{name of the set})}$: is the membership function of particular set
5. $\mu_{(\text{name of the set})}(T)$: is the degree of membership for any given fuzzy set.

Membership function and its degree of participation:

$$\mu_{Low}(T) = \begin{cases} 0 & T < low \text{ and } T > high \\ 0 < \text{and} < 1 & (T > low \text{ and } T < cen) \\ & \text{OR } (T > cen \text{ and } T < high) \\ 1 & T = cen \end{cases}$$

where *low* = -5 and *high* = 15;

$$cen = \frac{\text{width}}{2} = \frac{-5+15}{2} = 5;$$

$$\mu_{Medium}(T) = \begin{cases} 0 & T < low \text{ and } T > high \\ 0 < \text{and} < 1 & (T > low \text{ and } T < cen) \\ & \text{OR } (T > cen \text{ and } T < high) \\ 1 & T = cen \end{cases}$$

where *low* = 5 and *high* = 25;

$$cen = \frac{\text{width}}{2} = \frac{5+25}{2} = 15;$$

$$\mu_{High}(T) = \begin{cases} 0 & T < low \text{ and } T > high \\ 0 < \text{and} < 1 & (T > low \text{ and } T < cen) \\ & \text{OR } (T > cen \text{ and } T < high) \\ 1 & T = cen \end{cases}$$

where *low* = 15 and *high* = 35;

$$cen = \frac{width}{2} = \frac{15+35}{2} = 25;$$

7. Fuzzy Logic in Clustering

Clustering is the unsupervised method [9] wherein records are grouped together and a discovery tool for data analysis that solves classification problems, such as effective partitioning of a large data set into homogeneous groups or clusters. Its objective is to distribute cases like people, objects, events etc. into groups, so that the degree of association to be strong between members of the same cluster and weak between members of different clusters [10]. It is often used when little is known about the distribution of data [10]. For the information which is unknown or not absolute, we can use the fuzzy logics for the predictions. So fuzzy logic provide a better solution to classify the data and provide a more natural decision which matches the real world situations more perfectly. In our paper on clustering, we used several techniques to classify the data like hierarchical [12], non-hierarchical which are further divided into Agglomerative technique, devisive technique and *k*- Means Clustering techniques etc [10]. Here we apply fuzzy logic with simple devisive technique for detection and to reach at a final decision.

Algorithm:

Step 1: U is the universe of disclosure which contains all the elements i.e.

U := {X_1, X_2, .. X_n) just like a single large cluster CK [i] such that CK [i] :={X} where X := (X_1, X_2, .. X_n)[10] which we have to devise into smaller sub-clusters/ fuzzy sub-sets according to matching degree defined by membership functions of different fuzzy sets.

Step 2: Input the number of clusters/fuzzy sets formed, let us define by the variable *t_clus*.

Step 3: Sort the elements: *i* := 0;

```
while (i < n-1) do
{
For j := i+1  to  n-1 do //(j := i+1; j<n; j++)
  {
     If (CK [i] > CK [j]) then
{temp := CK[i];
CK [i] := CK [j];
CK [j] := temp;}; i++;
  }
}
```

Step 4: calculate the *low* and *high* for the boundaries of different clusters/fuzzy sets so that we can define the Rules for the membership functions and then calculate the mean/center of each cluster/fuzzy-set to find the membership degree: - $\mu_{(\text{Fuzzy set})}(\text{input})$.

Low [j], High [j] are arrays, which store the *low* and *high* of different sets (lowest and highest element) and Clust [j] is the array of clusters finally created.

//To calculate *low* and *high*:
If (n / *t_clus* := 0) then

```
limit := n / t_clus;
else limit := n / t_clus+1;
count := 2;
i := 0; low := 0;
while (i > n) do
{
j := 1;
while (j <= t_clus) do
{
Low [j] :=CK [low]; // to store the lowest element (low)
          of different sets
      If (i  == limit) then
   {
High [j] := CK [limit]; // to store the highest element
          (high) of different sets
          limit := limit * count;
          count := count +1;
      j++;}
    }
  low := limit+1;
  i++;
  }
```

Step 5://to put the elements into different clusters:

```
j := 1, lmt := 0;

while (j <= t_clus) do
{
For i := lmt  to n-1 do  // I=0 ;I<n; I++
{
Clust [j, i] := CK[i]; // Clust [j][i] := CK[i];
if(High[j]  == CK[i]) then
{j := j+1;// for the next cluster
    lmt := i;}}
}
```

Step 6: //find the membership degree $\mu_{(set)}(T)$ where set is the name of set and T is the input variable.

```
For i := 1 to t_clus do  // i := 1; i <= t_clust; i++
{
//cen is a variable which store the center of a particular
set such that:
```

$$cen := \frac{Low [i] + High [i]}{2};$$

```
//define the fuzzy rule for each cluster/ fuzzy set
Rule [i, 1]:     If (T < Low [i] and T > High [i]) then
                 // Rule [i][1]
                 μ(Clust [i])(T) := 0;
Rule [i, 2]:     If ((T > Low [i] and T < cen) or
                 (T  > cen and T < High [i]) ) then
                 0 <  μ(Clust [i])(T) < 1;
Rule [i, 3]:     If (T  == cen) then
                 μ(Clust [i])(T) := 1;
}
```

Step 7: Compare the degree of input variable with the predefined Rules and infer- to which set this variable belongs to and give the output. The fuzzy outputs for all rules are finally aggregated to one fuzzy set. To obtain a crisp decision from this fuzzy output, we have to de-fuzzify the fuzzy set, or the set of singletons [2]. Therefore, we have to choose one representative value (to take the center of gravity [2] of the each fuzzy set i.e. center) as the final output by applying any of the heuristic methods (de-fuzzification methods). Then we can compare the input linguistic variable to this representative value (center) and compute the minimum distance from the center using Euclidean distance measure [11], [12], [13].

Step 8: We can also graphically design a particular system using fuzzy logic as we do in our example of temperature (Fig.10); where we show the degree of participation of each input by membership function (μ), from a universe of disclosure U.

The problem of algorithm is that if a variable is at the point of intersection of two sets (.5 on y-axis in Fig.10), then in what set should we include that variable is still a matter of thinking because at this point, the input variable has the same degree in both the intersecting sets, like input 10 on x-axis has the same degree .5 in both the sets Low and High in Fig. 10 and 20 in Figs. 8.

Conclusions and Further Scope of work

In the present paper, fuzzy logic with its meaning and applications has been discussed and further defined as how fuzzy logic helps in providing a realistic approach in solving the real world problems, which resembles the human intelligence more perfectly. Fuzzy logic is capable of providing precise responses. It allows systems built around Boolean logic, handling binary values as well as to work with imprecisely defined values that you might express verbally as "more," "less," "high," "low," and so on. It is a fine technique where our decisions are based on guessing rather than accurate and complete data. Although vague, those data values are amenable to intuitive human understanding. We also presented the utilisation of fuzzy logic in clustering technique of data mining.The scope of fuzzy logic is wide, as we can use this not only in data mining techniques but also in other areas, like in scientific formulas, business management, mathematical approaches, in designing the systems (small as well as complex) i.e. Control systems (robotics, manufacturing and process control, automation, tracking, and consumer electronics), information systems, pattern recognition and other many areas. Our topic for the further study may be the neural networks which, unlike fuzzy logic, seek to reproduce the versatility of the human brain in recognizing the end-to-end, input-to-output behavior of a system without understanding all the processes taking place within it. Neural networks have the attributes of memory & learning and are therefore particularly applicable to complex problems involving difficult-to-measure or difficult-to-model parameters—in particular, processes that have a high degree of nonlinearity.

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