EHD Analysis of a Four-Lobe Pressure Dam Bearing with Rigid and Flexible Rotors

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Abstract: *It is observed that among multi-lobe pressure dam bearings, a four-lobe bearing pressure dam possesses good stability characteristics. A four-lobe pressure dam bearing is produced by cutting two pressure dams on the upper two lobes and two relief-tracks on the lower two lobes of an ordinary four-lobe bearing. The elastohydrodynamic analysis of a four-lobe pressure dam bearing has been described in this paper. The results indicate that liner flexibility considerably affects the stability of a four-lobe pressure dam bearing. The minimum threshold speed decreases whereas the zone of infinite stability increases with increase in the liner flexibility. The rotor flexibility aversely affects the minimum threshold speed but the zone of infinite stability remains unaffected.*

1. NOMENCLATURE

c	: radial clearance
c_m	: minimum film thickness for a centered shaft
$C_{xx}, C_{xy}, C_{yx}, C_{yy}$	: oil-film damping coefficients
$\bar{C}_{xx}, \bar{C}_{xy}, \bar{C}_{yx}, \bar{C}_{yy}$	: dimensionless oil-film damping coefficients, $\bar{C}_{xx} = C_{xx}(\omega c/W)$
C_0, C_1, C_2, C_3, C_4	: coefficients of the characteristic equation
D	: diameter
e	: eccentricity
F	: dimensionless shaft flexibility, W/ck
h	: oil-film thickness, $c(1+\varepsilon\cos\theta)$
$\bar{h}$	: dimensionless oil-film thickness, $h/2c$
$2k$	: shaft stiffness
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	: oil-film stiffness coefficients
$\bar{K}_{xx}, \bar{K}_{xy}, \bar{K}_{yx}, \bar{K}_{yy}$	: dimensionless oil-film stiffness coefficients, $\bar{K}_{xx} = K_{xx}(c/W)$
L	: bearing length
N	: journal rotational speed
O_i	: lobe center of lobe i (i = 1,2,3,4)
p	: oil-film pressure
R	: journal radius
t_h	: thickness of the bearing liner (L)
S	: Sommerfeld number, $\frac{\mu NLD}{W}\left(\frac{R}{c}\right)^2$
V	: peripheral speed of journal
W	: bearing external load
x, z	: coordinates for bearing surface (x-peripheral, z-along shaft axis)
ϕ	: attitude angle
α	: whirl rate ratio, $\alpha = \dot{\phi}/\omega$
β	: squeeze rate ratio, $\beta = \dot{\varepsilon}/\omega$
ε	: eccentricity ratio, e/c
δ	: ellipticity ratio, $(1 - c_m/c)$
θ	: angle measured from the line of centers in the direction of rotation
θ_g	: oil-groove angle
ρ	: fluid density
μ	: average fluid viscosity

ω : rotational speed

ν : dimensionless threshold speed, $\omega(c/g)^{1/2}$

ψ : deformation coefficient, $\frac{\mu\omega}{E}\frac{t_h}{R}\left(\frac{R}{c}\right)^3$

g : gravitational acceleration constant

2. INTRODUCTION

The current trend in the industry is to run the turbomachines at high speeds. The ordinary circular bearings, which are the most common type of the bearings, are found to be unstable at high speeds. It is found that the stability of these bearings can be increased by the use of multilobes and the incorporation of pressure dams in the lobes. The analysis of multi-lobe bearings was first published by Pinkus [1]. It was followed by Lund and Thomson [2] and Malik [3] et al., who gave some design data which included both static and dynamic characteristics for laminar, as well as turbulent flow regimes. The experimental stability analysis of such types of bearing [4]-[5] showed that the analytical stability analysis reflects the general trends in experimental data. The analytical dynamic analysis has shown that non-cylindrical pressure dam bearings [6]-[9] are found to be very stable. The elastohydrodynamic (EHD) studies of some ordinary and pressure dam bearings [10]-[12] have indicated that flexibility of the bearing liner affects the performance of the bearing. No work has been reported on the EHD analysis of a four-lobe pressure dam bearing. Therefore the effect of liner flexibility on the performance of a four lobe pressure dam bearing with rigid and flexible rotor has been studied.

3. BEARING GEOMETRY

The geometry of a four-lobe pressure dam bearing is shown in Figure 1. A four-lobe pressure dam bearing comprises four lobes whose centers of curvatures are not in the geometrical center of the bearing. Thus, though the individual lobes are circular, the geometrical configuration as a whole is not. A rectangular dam or step of depth S_d and width L_d is cut circumferentially in each of the lobes 1 and 4. The dam starts after the oil hole and subtend arc of θ_s degrees at the center. Circumferential relief-tracks or grooves of certain depth and width L_t are also cut centrally in the lobes 2 and 3 of the bearing. The relief-tracks are assumed to be so deep that their hydrodynamic effects are neglected. For concentric position of the rotor, there are two reference clearances of the bearing: a major clearance c given by a circle circumscribed by the lobe radius and a minor clearance c_m given by an inscribed circle. Thus the center of each lobe is shifted by a distance $e_p = (c - c_m)$ known as ellipticity of the bearing. The various eccentricity and ellipticity are non-dimensionalized by dividing the major clearance c.

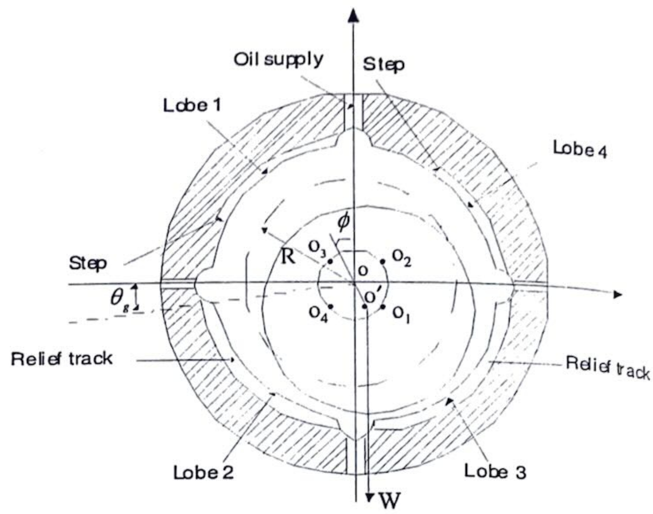

Fig. 1: A four-lobe pressure dam bearing.

Ellipticity ratio $(\delta) = (c - c_m)/c = 1 - c_m/c$

Eccentricity ratio $(\varepsilon) = e/c$

$\varepsilon_1 = e_1/c, \varepsilon_2 = e_2/c, \varepsilon_3 = e_3/c, \varepsilon_4 = e_4/c$

The various eccentricity ratios and attitude angles of the lobes of a four-lobe pressure dam bearing are given by:

$$\varepsilon_1^2 = \varepsilon^2 + \delta^2 - 2\varepsilon\delta\cos(\pi/4 - \phi)$$
$$\varepsilon_2^2 = \varepsilon^2 + \delta^2 + 2\varepsilon\delta\sin(\pi/4 - \phi)$$
$$\varepsilon_3^2 = \varepsilon^2 + \delta^2 + 2\varepsilon\delta\sin(\pi/4 + \phi)$$
$$\varepsilon_4^2 = \varepsilon^2 + \delta^2 - 2\varepsilon\delta\cos(\pi/4 + \phi)$$

$$\phi_1 = 5\pi/4 + \gamma + \sin^{-1}(\varepsilon(\sin\pi/4 - \phi)/\varepsilon_1)$$
$$\phi_2 = 7\pi/4 + \gamma + \sin^{-1}(\varepsilon(\cos\pi/4 - \phi)/\varepsilon_2)$$
$$\phi_3 = \pi/4 + \gamma - \sin^{-1}(\varepsilon(\cos\pi/4 + \phi)/\varepsilon_3)$$
$$\phi_4 = 3\pi/4 + \gamma - \sin^{-1}(\varepsilon(\sin\pi/4 + \phi)/\varepsilon_4)$$

4. ANALYSIS

The non-dimensionalized Reynolds Equation for the laminar flow is:

$$\frac{\partial}{\partial \bar{x}}\left(\frac{\bar{h}^{3}}{12}\frac{\partial \bar{p}}{\partial \bar{x}}\right)+\left(\frac{D}{L}\right)^{2}\frac{\partial}{\partial \bar{z}}\left(\frac{\bar{h}^{3}}{12}\frac{\partial \bar{p}}{\partial \bar{z}}\right)=\frac{\pi}{2}\frac{\partial \bar{h}}{\partial \bar{x}}+$$

$$\pi\varepsilon\dot{\alpha}\sin 2\bar{x}+\pi\dot{\beta}\cos 2\bar{x} \qquad (1)$$

The solution of the above equation for pressure distribution in the finite element technique is obtained by minimizing the following variational integral [13] over the individual elements:

$$J_e(\bar{p}_e)=\iint_{A_e}\left[-\frac{1}{2}\frac{\bar{h}^3}{12}\left\{\left(\frac{\partial \bar{p}_e}{\partial \bar{x}}\right)^2+\left(\frac{D}{L}\right)^2\left(\frac{\partial \bar{p}_e}{\partial \bar{z}}\right)^2\right\}+\frac{\pi}{2}\bar{h}\frac{\partial \bar{p}_e}{\partial \bar{x}}-\right.$$

$$\left.\frac{\pi}{2}\varepsilon\dot{\alpha}\cos 2\bar{x}\frac{\partial \bar{p}_e}{\partial \bar{x}}+\frac{\pi}{2}\dot{\beta}\sin 2\bar{x}\frac{\partial \bar{p}_e}{\partial \bar{x}}\right]dA_e \qquad (2)$$

where $\bar{p}_e$ = dimensionless film pressure in the eth element.

Each lobe of the bearing is analysed separately. Since the pressure profiles for the lobes 1 and 4 are symmetrical about the center line of the bearing, only half of the lobe is considered for analysis. The mesh size is reduced near the trailing edge where heavy pressures are produced. The resulting matrix in each case is stored in the banded form and is then solved by the Gauss-elimination method.

The Reynolds equation is an elliptical partial differential equation and, hence must be solved as a boundary value problem. According to MaCallion, *et al* [14], for a bearing having oil supplied at zero pressure, the largest possible extent of the positive pressure region is given by the boundary conditions that both pressure and pressure gradients are zero at the breakdown and build up boundaries of the oil-film. However, it has been shown [15] that even by setting the negative hydrodynamic pressures to zero as they occur in any iteration step, the results tend to satisfy the above mentioned boundary conditions in the limit. The latter approach has been followed in the present analysis. Stiffness and damping coefficients are calculated separately for each lobe and then totaled. The values of these stiffness and damping coefficients, shaft flexibility, and dimensionless speed are then used to evaluate the coefficients of the characteristic equation , which is a polynomial of the 6th order for flexible rotors[16].

The characteristic equation is:

$$s^6F^2\nu^4C_0+s^5\nu^4\left(F^2C_1+FC_2\right)+$$
$$s^4\nu^2\left(\nu^2F^2C_3+2FC_0+\nu^2+\nu^2FC_4\right)$$
$$+s^3\nu^2\left(2FC_1+C_2\right)+s^2\left(2F\nu^3C_3+\nu^2C_4+C_0\right)+$$
$$sC_1+C_3=0 \qquad (3)$$

For a rigid rotor, the value of F is taken as zero. The system is considered as stable if the real part of all roots is negative. For a particular bearing geometry and eccentricity ratio, the values of dimensionless speed are increased until the system becomes unstable. The maximum value of speed for which the bearing is stable is then adopted as the dimensionless threshold speed.

Solution of a bearing problem considering the flexibility effects of the liner involves the simultaneous solution of Reynolds equation in the lubricant and the linear elasticity equations in the bearing liner. While finding the solution, it is assumed that the liner is enclosed in a rigid housing and the thermal effects are neglected. The deformation of the bearing liner is obtained by using a three-dimensional elasticity model using hexahedral elements. To account for the flexibility of the bearing liner, a dimensionless deformation coefficient as a function of the journal speed, geometry of the bearing, viscosity of the lubricant, thickness of the bearing liner and the modulus of the elasticity of the liner material is defined. The finite element formulation of the model is given by the equation:

$$[K_e]\{\bar{\delta}_e\}=\psi\{\bar{F}_e\}$$

where, $\psi=\frac{\mu\omega}{E}\frac{t_h}{R}\left(\frac{R}{c}\right)^3$

Since it assumed that the bearing liner is enclosed in a rigid housing, the displacements at all the nodal points of the liner, which are in contact with the housing are made zero. As symmetry is considered and only half of the bearing structure is analysed. The nodes at the mid plane have zero displacements in the axial direction. At the solid-fluid interface, sides of the hexahedral elements form a network of quadrilateral elements, which becomes the finite element network in the fluid film. Thus, the nodal displacements of the solid can be directly related to the changes in the nodal film thickness and the nodal pressures of the lubricant can be directly correlated with external nodal forces acting on the solid surface.

The present analysis has been done for the bearing with the following parameters which have been optimized for the best stability [17].

L/D = 1, = 1.0, = 0.8, = 0.25, = 55°, =0.5

To study the effect of liner flexibility on the performance of the four-lobe pressure dam bearing, the values of the considered are 0, 0.02, 0.05 and 0.1.The values of the rotor flexibility F selected for the study are 0, 0.5, 1.0, 2.0 and 4.0.

5. RESULTS AND DISCUSSION

The effect of liner flexibility on the stability of a four-lobe pressure dam bearing supporting a rigid rotor is shown in Figure 2.

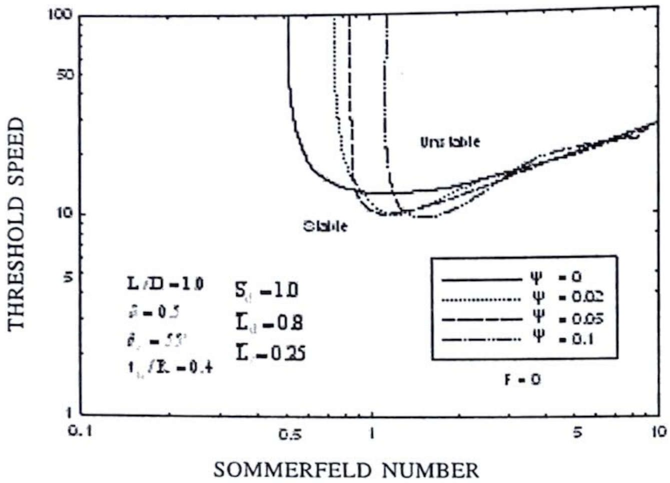

Fig. 2: Effect of liner flexibility on the stability of a four-lobe pressure dam bearing with a rigid rotor (F=0).

The plots of the figure depict that with increase in the flexibility of the liner, the value of minimum threshold speed decreases. However, the zone of infinite stability is observed to increase with increase in the liner flexibility. The values of minimum threshold speed and zone of infinite stability at the various deformation coefficients are shown in Table 1 below.

Table 1

Deformation coefficient (ψ)	Minimum Threshold speed	Zone of infinite stability (up to S)
0	12.8	0.5
0.02	10.05	0.73
0.05	10.0	0.85
0.1	9.65	1.18

The effect of liner flexibility on the stability of a four-lobe pressure dam bearing with flexible rotors is shown in Fig. 3 to 6.0. The same effects are observed for a flexible rotor as for a rigid rotor. The minimum threshold speed decreases, whereas, zone of infinite stability increases with increase in liner flexibility. However, at higher values deformation coefficient (ψ = 0.05 and 0.1), the decrease in the minimum threshold speed is marginal while the gain in zone of infinite stability is substantial. It is also observed from these plots that as the rotor flexibility is increased from 0.5 to 4.0, for a particular value of, the minimum threshold speed reduces while the zone of infinite stability remains unchanged. Thus, in general with increase in the rotor flexibility the stability of the bearing decreases.

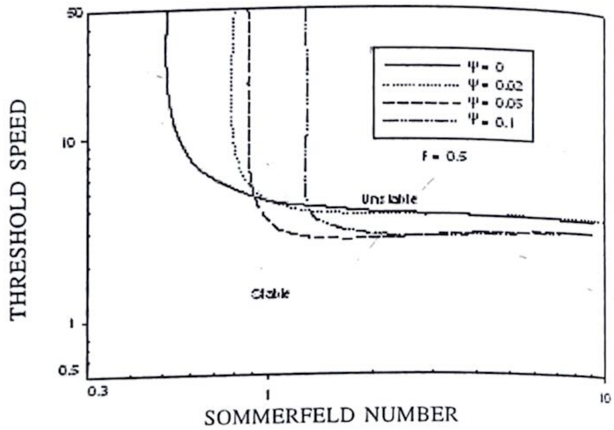

Fig. 3: Effect of liner flexibility on the stability of a four-lobe pressure dam bearing with a flexible rotor (F=0.5).

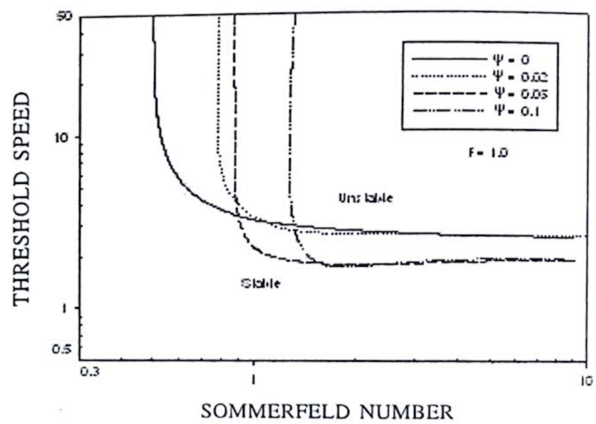

Fig. 4: Effect of liner flexibility on the stability of a four-lobe pressure dam bearing with a flexible rotor (F=1.0).

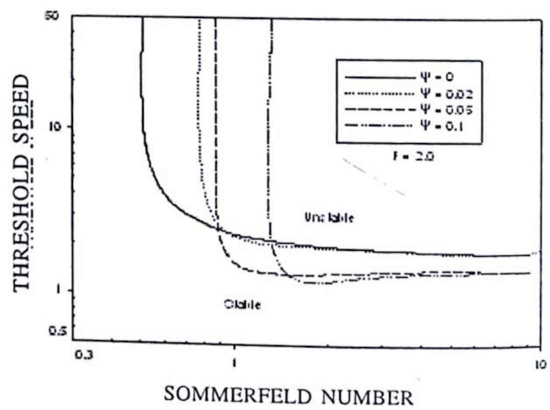

Fig. 5: Effect of liner flexibility on the stability of a four-lobe pressure dam bearing with a flexible rotor (F=2.0).

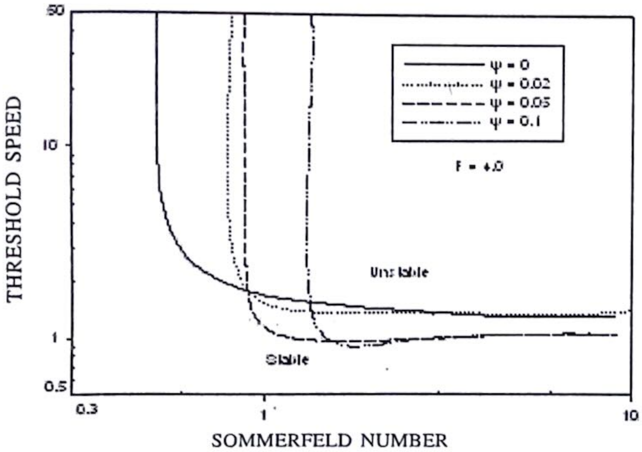

Fig. 6: Effect of liner flexibility on the stability of a four-lobe pressure dam bearing with a flexible rotor (F=4.0).

6. CONCLUSIONS

1. The liner flexibility affects the stability of a four-lobe pressure dam bearing with rigid as well as flexible rotor. The minimum threshold speed decreases whereas the zone of infinite stability increases with increase in liner flexibility.

2. The increase in rotor flexibility adversely affects the minimum threshold speed but the zone of infinite stability remains unchanged.

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