Study of Human Joint Under Highly Loaded Conditions: Effect of Exponential Slider Bearing

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Abstract: *A theoretical analysis of the lubrication mechanism occurring in human knee joint is presented. The Reynolds equation is derived to account for the sliding effect. The pressure and load capacity of bearing system are obtained when the film thickness is an exponential function. In this paper, the effects of HA molecules in the synovial fluid of human knee joint are studied under highly loaded conditions. The effect of viscosity variation is taken into account for both normal and diseased joints. It is found that as the parameter of viscosity i.e., the concentration of HA molecules increases, the load bearing capacity of normal human joint also increases and of diseased joint decreases.*

Keyword: *Reynolds' equation,exponential slider bearing, Hyaluronic acid (HA) molecules, Lubricant flux, Load capacity.*

I. INTRODUCTION

The theory of lubrication flow has been in use for well over a century, beginning with Tower's 1883 experiments of a rolling bearing [1] and Osborn Reynolds formulated a theory of lubrication in 1886 [2] Since then, Reynolds' theory has been the foundation of the theory of hydrodynamic lubrication. Recent books by Hori [3] and Szeri [4] thoroughly review the history and applications of lubrication theory and derive the governing equations.

The classical analysis of the hydrodynamic lubrication of slider bearings was presented by Pinkus and Sternlicht [5]. Exact solutions of Reynolds' equation for slider bearings with various simple film geometries were discussed in a number of books and research papers (Cameron [6], Archibald [7], Lord Rayleigh [8], Charnes and Saibel [9], Basu, Sengupta and Ahuja [10], Majumdar [11], Hamrock [12], Gross, Matsch, Castelli, Eshel, Vohr and Wildmann [13]). Prakash and Vij [14],Bhat [15] analyzed the hydrodynamic lubrication of a plane slider bearing taking different geometries into consideration and also analysed an inclined plane slider bearing and an exponential slider bearing respectively. Use of porous matrix decreased the load capacity and friction force on the slider. Cameron [16] suggested that an exponential form of the slider to be nearest the true shape.

Hays [17] studied the effect of surface curvature on the squeeze films for rectangular plates whereas he suggested that the geometry of the surface should be held to close tolerance if loads are to be sustained which creates thin squeeze films.Tichy et al. [18] made an attempt to study inertial considerations in parallel circular squeeze films bearings.Qvale et al. [19] pointed out that the coefficient of friction can be reduced or increased significantly for a given geometry, load and film thickness if the viscosity varies in the direction perpendicular to the direction of motion.Shukla et al. [20] derived a generalised form of Reynolds equation for fluid lubrication by considering the effects of viscosity variation in the film and slip at the bearing surfaces. They also introduced the concept of multiple-Layer lubrication.Tandon et al. [21] studied temperature regulation in synovial joints and observed that the rise in temperature is more in the osteo- arthritic synovial fluid as compared to normal and young synovial fluids.Shah et al. [22] gave an analysis of a porous exponential slider bearing lubricated with a ferrofluid considering slip velocity at the porous interface.

Naduvinamani et al. [23] investigated the combined effects of unidirectional surface roughness and the lubricant additives on the performance characteristics of porous squeeze film lubrication between two rectangular plates of finite dimensions.Bali et al. [24] proposed a non- linear model for lubrication in synovial joints whereas they modelled the cartilage by mixture of two distinct constitutes i. e. an incompressible fluid phase and an incompressible porous solid phase.Han et al. [25] studied the hydrodynamic lubrication properties of the textured surface with asymmetric microdimple and revealed that the asymmetric microdimple can obtain large load- carrying capacity than a symmetrical rectangular microdimple.Deheri et al.[26] made an attempt to analyse the squeeze film behaviour between rough porous rectangular plates with the surfaces transversely rough and suggested that while designing the bearing system the roughness must be given due consideration.Roberts et al. [27] gave an extension of Reynolds lubrication theory to model the effect of free fluid interfaces and fluid-structural interactions in lubrication films and by simulations using this model are compared to analytical solutions and experimental results for a number of model problems.

The synovial joints, provided by nature in the human body to carry out trouble free motion of one bone past another, have long been identified as a bearing system. In fact these joints function as excellent bearings in biological conditions. The study of mechanism of synovial joints has recently become an active area of scientific research. The human joint is a dynamically loaded bearing which employs articular cartilage as the bearing and synovial fluid as the lubricant. Once a fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The loaded bearing synovial joints of the human body are the shoulder, hip, knee, and ankle joints.

The present paper is a theoretical analysis of the lubrication mechanism occurring in human knee joint for an exponential slider bearing case and the film thickness is an exponential function. During walking cycle, the curvature of the joint surfaces does play an important role in producing the wedge film action similar to hydrodynamic lubrication theory, McConail [28]. We consider the effects of sliding under walking condition. Since the time taken by a complete walking cycle is the order of only one second, Mow et al. [29]

II. MATHEMATICAL MODEL

In the present study we have modelled the function of the knee joint under highly loaded conditions such that HA molecules make a three dimensional network in the synovial fluid forming a porous matrix which creates resistance to flow of plasma in the fluid. The equations governing the parallel flow of plasma in the film gap is given as follows (in the fixed coordinate system with respect to the origin). A geometrical situation is shown in Fig 1.

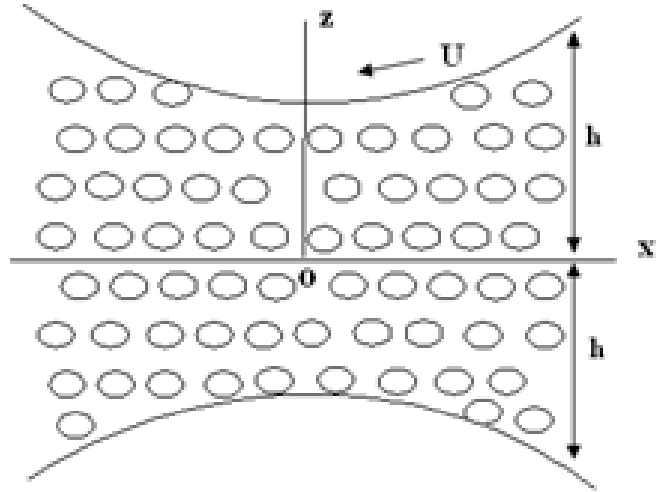

Fig. 1: Geometrical representation of the contact zone of the single layer case

Region: $0 \leq Z \leq h$

$$\frac{\delta}{\delta z}\left(\mu \frac{\delta u}{\delta z}\right) = \frac{\delta p}{\delta x} \qquad \ldots 1$$

Where u is the velocity in the x-direction in the film gap. The coefficient μ is the viscosity of blood plasma in the film; 2h is the total thickness of the fluid film, which may be a function of x (Fig.1).

Boundary conditions are

$$u = -U \text{ at } Z = h \qquad \ldots 2$$

$$\frac{\delta}{\delta z} = 0 \text{ at } z = 0 \qquad \ldots 3$$

Normal Human Knee Joint: In the present work the viscosity is variable and is a function of z and also assume that the HA molecules are accumulated in synovial joint. i.e.

$$\mu = \mu_0 \exp\left\{\beta\left(\frac{z}{h}\right)\right\} \qquad \ldots 4$$

Where β is a parameter of viscosity.

Solving Equation (1) using the boundary conditions (2), (3) and Equation (4), we have –

$$u = \left(\frac{\delta p}{\delta x}\right) \frac{h}{(-\beta)\ \mu_0\ (\exp(-\beta) -1)} \left\{\exp\left(\frac{\beta z}{x}\right)\ (z \exp(-\beta) - h \exp(-\beta) - z) + \exp(-\beta)\right\} + \frac{U}{(\exp(-\beta) -1)} \left\{\exp\left(\frac{\beta z}{h}\right) - \exp(-\beta)\right. \quad ...5$$

The total lubricant flux Q per unit width can be defined as –

$$\frac{Q}{Z} = \int_O^h u\, dz \quad ...\ 6$$

Which on using the value of u from equation (5), gives

$$Q = -\left(\frac{\delta p}{\delta x}\right) [F_{11}] - U\ [G_{11}] \quad ...\ 7$$

Where

$$F_{11} = \frac{2h^3}{(-\beta^3)\ \mu_0\ (\exp(-\beta) -1)} \{\exp(-\beta)\ (\exp(-\beta) - 2 - \beta^2) + 1\} \quad ...\ 8$$

$$G_{11} = \frac{2h}{(-\beta^3)\ (\exp(-\beta) -1)} \{1-(1+\beta) \exp(-\beta)\} \quad ...\ 9$$

After integrating the continuity equation

$$\frac{\delta u}{\delta x} + \frac{\delta w}{\delta z} = 0$$

across the two layers for $z \geq 0$ (i.e. from 0 to h) and using the following conditions for vertical velocity w.

$w = 0$ at $z = 0$

$w = 0$ at $z = h$

We get,

$$\frac{d}{dx}\ (Q) = 0 \quad ...\ 0$$

From equations (7) and (10), the differential equation governing the pressure can be obtained as–

$$\frac{d}{dx}\left(-F_{11} \frac{\delta p}{\delta x}\right) = u\ \frac{\delta}{\delta x}\ (G_{11}) \quad ...11$$

The above equation takes account of the pure rolling case (running or walking position).

III. PURE ROLLING CASE

Here we consider the effect of sliding of synovial fluid under jumping condition. During jumping, the curvature of the joint produces the wedge film action in the direction of motion due to elasticity of cartilage. The film thickness h may be written as,

$$h = h_0 \exp(\alpha\ x)$$

Where h_0 is the minimum film thickness, á is the averageinclination of the curved surface from the horizontal and is assumed to be very small. The geometrical situation isshown in Fig. 2.

$$á \approx \tan(á) = (h-h_0) / l\ ,\ á\ R \approx l$$

$$á \approx (h - h_0) / l \approx l/R$$

As shown in Fig. 2, á has the following approximaterelationwith the conforming R of the joint and it decreases as Rincreases.

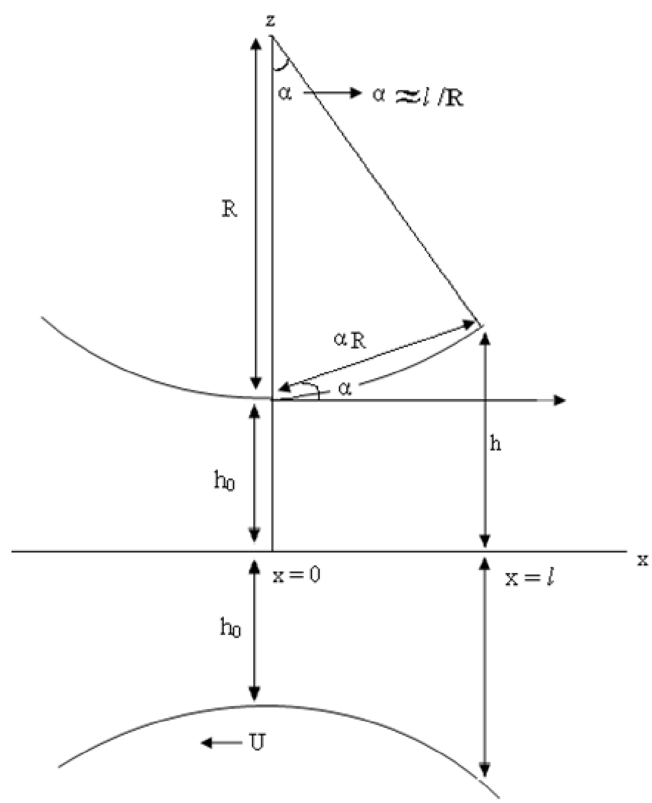

Fig. 2: Effect of sliding in the synovial joint

$a \approx \frac{1}{R}$ (<< 1)

Boundary conditions for pressure are:

$p = 0$ at $x = 0$ ...12

$p = 0$ at $x = 1$ ...13

Using above boundary conditions in equation (11), we get–

$$\frac{\delta p}{\delta x} = -\frac{u\mu_0\beta^2 \{1-(1+b)\exp(-b)\}}{(h_0)^2 \{\exp(-\beta)(\exp(-\beta)-2-\beta^2)+1\}}$$

$$\left\{\exp(-2\alpha x) - \exp(-3\alpha x)\frac{\int_0^1 \exp(-2\alpha x)\,dx}{\int_0^1 \exp(-3\alpha x)dx}\right\}$$

The Load capacity is given by–

$$W_{11} = \frac{2bU\mu_0 \qquad \beta^2\{1-(\beta+1)\exp(-\beta)\}}{(h_0)^2\{\exp(-\beta)(\exp(-\beta)-2-\beta^2)+1\}}$$

$$\left[\int_0^1 \left\{\exp(-2\alpha x) - \exp(-3\alpha x)\frac{\int_0^1 \exp(-2\alpha x)\,dx}{\int_0^1 \exp(-3\alpha x)dx}\right\} x\,dx\right]$$

Where b is the width of the contact zone

In the non-dimensional form W_{11} can be written as–

$$\overline{W}_{11} = \frac{W(h_0)^2}{2bU\mu 0l^2} = \frac{\beta^2\{1-(\beta+1)\exp(-\beta)\}}{\{\exp(-\beta)(\exp(-\beta)-2-\beta^2)+1\}}$$

$$\left[\int_0^1 \left\{\exp(-2\bar{\alpha}\bar{x}) - \exp(-3\bar{\alpha}\bar{x})\frac{\int_0^1 \exp(-2\bar{\alpha}\bar{x})\,d\bar{x}}{\int_0^1 \exp(-3\bar{\alpha}\bar{x})d\bar{x}}\right\} \bar{x}\,d\bar{x}\right]$$

Where $\bar{\alpha} = \alpha l$, $\bar{x} = x/l$

Diseased Human Knee Joint: In the present work the viscosity is variable and is a function of z and also assume that the HA molecules are not accumulated in synovial fluid, i.e.

$$\mu = \mu_0 \exp\left\{-\beta\left(\frac{z}{h}\right)\right\} \quad \text{... 14}$$

Solving Equation (1) using the boundary conditions (2), (3) and Equation (14), we have–

$$u = \left(\frac{\delta p}{\delta x}\right)\frac{h}{\beta\mu_0(\exp(\beta)-1)}\left\{\exp\left(\frac{\beta z}{h}\right)(z\exp(\beta) - h\exp(\beta) - z) + h\exp(\beta)\right\} + \frac{h}{(\exp(\beta)-1)}\left\{\exp\left(\frac{\beta z}{h}\right) - \exp(\beta)\right\} \quad \text{...15}$$

Using the Equation (15) in Equation (6), we get–

$$Q = -\left(\frac{\delta p}{\delta x}\right)[F_{12}] - U[G_{12}] \quad \text{...16}$$

Where

$$F_{12} = \frac{2h^3}{\beta^3\mu_0(\exp(\beta)-1)}\{\exp(\beta)(\exp(\beta)-2-\beta^2)+1\} \quad \text{...17}$$

$$G_{12} = \frac{2h^3}{\beta(\exp(\beta)-1)}\{\exp(\beta)(\beta-1)+1\} \quad \text{... 18}$$

From Equations (10) and (16), the differential equation governing the pressure can be obtained as–

$$\frac{d}{dx}\left(-F_{12}\frac{\delta p}{\delta x}\right) = U\frac{\delta}{\delta x}(G_{12}) \quad \text{...19}$$

Using the boundary conditions (12) and (13) in equation (19), we get–

$$\frac{\delta p}{\delta x} = -\frac{U\mu_0\beta^2\{\exp(\beta)(\beta-1)+1\}}{(h_0)2\{\exp(\beta)(\exp(\beta)-2-\beta^2)+1\}}$$

$$\left\{\exp(-2\alpha x) - \exp(-3\alpha x)\frac{\int_0^1 \exp(-2\alpha x)\,dx}{\int_0^1 \exp(-3\alpha x)dx}\right\}$$

The Load capacity is given by–

$$W_{12} = \frac{2bU\mu_0 \qquad \beta^2\{\exp(\beta)(\beta-1+1\}}{(h_0)^2\{\exp(\beta)(\exp(\beta)-2-\beta^2)+1\}}$$

$$\left[\int_0^1 \left\{\exp(-2\alpha x) - \exp(-3\alpha x)\frac{\int_0^1 \exp(-2\alpha x)\,dx}{\int_0^1 \exp(-3\alpha x)dx}\right\} + x\,dx\right]$$

In the non-dimensional form W_{12} can be written as–

$$\overline{W}_{12} = \frac{W(h_0)^2}{2bU\mu 0l^2} = \frac{\beta^2\{\exp(\beta)(\beta-1)+1\}}{\{\exp(\beta)(\exp(\beta)-2-\beta^2)+1\}}$$

$$\left[\int_0^1 \left\{\exp(-2\bar{\alpha}\bar{x}) - \exp(-3\bar{\alpha}\bar{x})\frac{\int_0^1 \exp(-2\bar{\alpha}\bar{x})\,d\bar{x}}{\int_0^1 \exp(-3\bar{\alpha}\bar{x})d\bar{x}}\right\} \bar{x}\,d\bar{x}\right]$$

Where $\bar{\alpha} = \alpha l$, $\bar{x} = x/l$

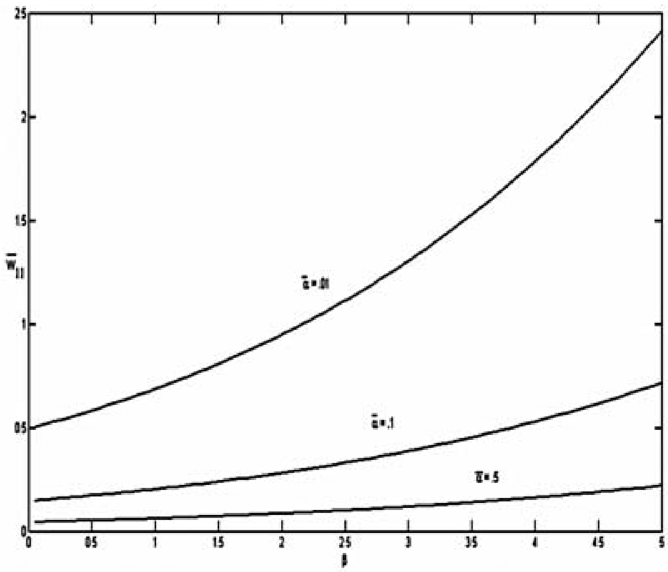

Fig. 3. Variation of $\overline{W_{11}}$ verses parameter of viscosity (β)

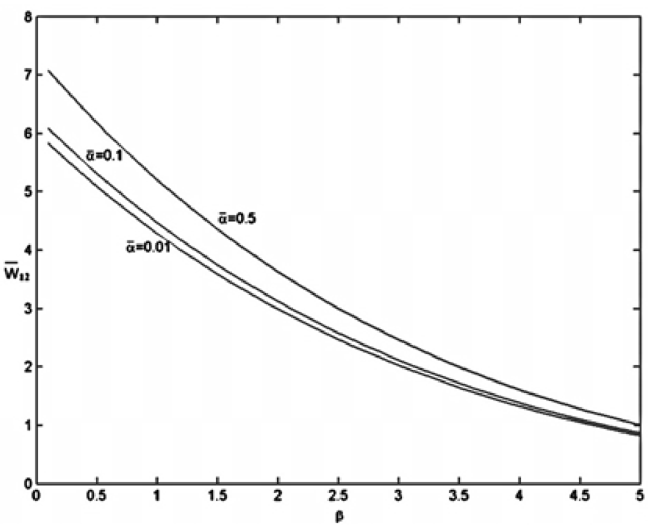

Fig. 4. Variation of $\overline{W_{12}}$ verses parameter of viscosity (β)

IV. CONCLUSION

In normal human knee joint when HA molecules are accumulated, Figure 3 shows that as the parameter of viscosity increases, load capacity increases. This implies that as the concentration of hyaluronic acid molecules in the gap increases, the load capacity of the normal joint increases but in diseased human knee joint when HA molecules are not accumulated, Figure 4 shows that as the concentration of hyaluronic acid molecules in the gap decreases, the load capacity decreases

V. ACKNOWLEDGEMENTS

The work of the first named author has been supported by Junior Research Fellowship of UGC (Maulana Azad National Fellowship for Minority Students) India, F1-17.1/2013-14/MANF-2013-14-BUD-UTT-25523/ (SA-III/Website).

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