Evolution of Contact Models for Different Types of Contacts

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Abstract: *Contact between two bodies may be considered as a point contact or a soft contact. In case of soft contact, both contact forces and moments exist over the contact area. In any contact model, deformation, contact area and distribution of contact forces over the contact area between the two surfaces are considered. After the development of Hertzian contact theory, a number of efforts have been made by many researcher to model contact mechanics of different types of contact. A fast and robust algorithm is required for contact detection between the two surfaces. A number of approaches to model friction and normal contact at the contact interface have been followed. This paper presents modeling approaches for different types of contact with their limitations, and it is concluded with the research gap and proposed contact models at the end.*

Keywords: *Contact Radius, Contact Interface,Finite Element Method, Modeling, Simulation,*

I. INTRODUCTION

Contact between two objects may be considered as a point contact or a soft contact. Point contact may be with or without friction. Point contact is simple to model but not realistic. In absence of friction, external force that acts on the bodies is normal force. When friction is considered, external forces at the point of contact can be resolved into normal and tangential direction as shown in Fig. 1(a).

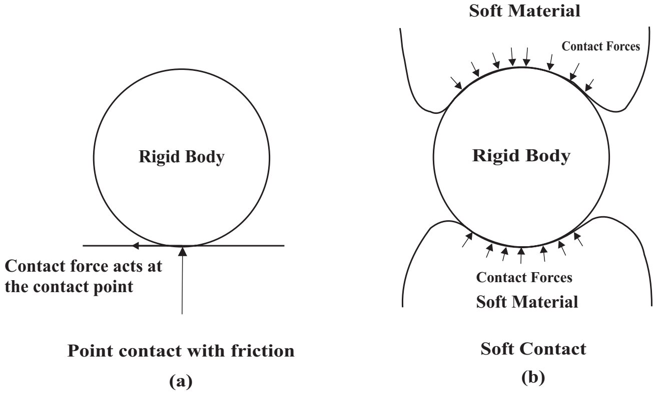

Fig. 1: (a) Point Contact with friction (b) Soft contact with contact forces distributed over contact area.

In case of point contact, no moments act on point of contact. In case of soft contact, contact area develops, and contact forces are distributed over it as shown in Fig. 1(b). Moments also develop overthe contact area which facilitates dexterous manipulation. In case of point contact, two collinear forces or minimum three forces are required to grasp an object as shown in Fig. 2. With point contact, object can't be manipulated efficiently.

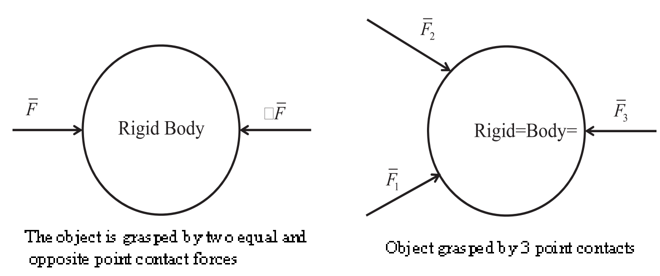

Fig. 2: Object grasping by contact forces

A number of efforts have been made to model contact dynamics for different types of contact. This paper presents different types of contact models, contact algorithm to detect contact region. A number of approaches were followed to model friction and normal contact which are presented in the subsequent section.

II. CONTACT MODELS

In 1881, Hertz Heinrich solved contact problem between two elastic spheres[1]. He determined that radius of contact is directly proportional to normal force raised to power 1/3 as given in Eq.(1). The model is applicable for small deformation between two linear elastic non-conformal surfaces. The model does not consider friction force that acts at the contact interface.

$$\alpha = \left[\frac{3LR}{4E^C} \right]^{1/3} \quad \ldots (1)$$

Where *a* is hertz contact radius, *L* is normal applied force, *R* and E^C are combined radius and *Young's Modulus of Elasticity.*Johnson-Kendal-Robert [2] considered the effect of surface energy on the contact between two elastic solids. They found that mechanical work is required to overcome adhesive force between two surfaces in intimate contact. Theyobserved that the contact area between two spherical bodies was considerably larger than given by Hertz contact model. It became constant at zero load and approached to Hertz contact area at high load as given in Eq. (2), and this model is popularly known as Johnson-Kendall-Robert (JKR) model.

$$\alpha = \left[\frac{3R}{4E^C} \left(L + 6\Upsilon\pi R + \sqrt{12\gamma\pi RL + (6\Upsilon\pi R)^2} \right) \right]^{1/3} \ldots 2$$

Where ϒ represents surface energy. For ϒ=0, in absence of adhesive force, contact radius is Hertz contact radius. Laterthe effect of van der Waals forces on contact between a ball and a plane was investigated, and it was found that these force increases elastic contact area [3]. These models are developed for linear elastic material.

Xydas and Kao [4] developed contact model for non-linear hemispherical soft finger which determines relation of contact area with the normal force. It was found that contact area is directly proportional to normal force raised to power which varies from 0 to 1/3. For linear elastic material γ =1/3 and for ideal soft material γ=0. They constructed friction limit surface on the basis of experimental results. Friction limit surface represents maximum limit of tangential force and moment at the contact point up to which slipping does not occur. It was found that rate of increase of contact area reduces as the normal for increases.Bakhy*et al.*[5] proposed a power law equations which determines contact width for hemicylindrical soft finger in contact with a plane for different applied load. It was found that contact area is directly proportional to normal force raised to power γ which varies from 0 to ½. For linear elastic material γ = 1/2. When contact area is maximum and further increase in the normal force does not change the contact area, γ = 0. Machado *et al.*[6] discussed a number of compliant contact models for multibody system dynamics.

It is observed that Hertzian contact theory is foundation for almost all contact models, but it is not applicable to impact contact as there is dissipation of energy during the impact. Most of the models are developed for simple geometries, and static contact. It is quite complex and challenging to calculate contact area and distribution of contact forces over it when contact interface moves during rolling or sliding of one body over the other. A number of efforts are made to develop contact theory for manipulative soft contact. In case of contact between a rigid body and a soft material, the properties of soft material affect the contact dynamics. Finite Element Method (FEM) has been used for modeling contact between the two surfaces.

Many researchers discretized continuously distributed properties and modeled the soft material using FEM.Namima*et al.*[7] simulated rolling of an object between two soft fingers using FEM and constrained stabilization method. The model is developed for planar case. Finite element method was used to animate human hand which interacts with the outside world[8]. It took more computation time. They suggested the use of boundary element method in which nodes only on the boundary are considered to model the contact. In these models, both the bodies are discretized. Vaz and Maini[9] modeled dynamics of soft contact deformation using bond graph with an advantage of FEM. They investigated the deformation of the soft material when it is subjected to different point load. Xydas*et al.*[10] analyzed two non-linear soft fingers using finite element method. Almost all the models focus on contact area and pressure distribution. Alinia*et al.*[11] considered a case of rigid cylinder rolling on the substrate system. They model tangential traction using three stick slip regime.Yamane and Nakamura[12] used stable penalty based approach to model frictional contact among a number of complex objects. They considered Coulombs frictional force in computer simulation. They developed iterative algorithm to calculate normal vector and depth of penetration at the point of contact of polygon object.

The contact can be modeled using three different approaches: (1) constrained based (2) impulse based

(3) penalty based approach. In constraint based approach, contact forces are determined by optimizing equality and inequality constraints, but the approach is useful for simple shape. Impulse based approach is more suitable for impact contact. In penalty based approach, both the surfaces are allowed to interpenetrate, and contact between two points is modeled using a spring. Contact force is the restoring force generated by the spring.Kucharski and Starzynski[13] studied the contact problem between a rough surface and a rigid flat plane theoretically and validated experimentally.Moisio*et al.*[14] used tactile sensors in a robotics hand; then measured contact area and position of the object. Different objects were held in robotics hand and gray grid image were obtain. These images measure intensity of pressure at different contact points. They also considered friction force at the contact interface.Ultrasonic technique was used to study contact between a wheel and a rail[15]. It was observed that real contact area strongly depend upon initial roughness of the surfaces in contact. A hemispherical soft finger was modeled on the basis of distribution of forces[16]. When the finger was deformed, its geometric aspect was studied. The material of the soft finger was assumed to be nonlinear. A contact force function was developed and validated experimentally.Hwang*et al.*[17] simulated a humanoid robot. A virtual spring and damper was used to model the contact between feet of robot and the floor. A robot which ascended the steep stair was considered. Normal force is the force generated by the spring and damper system. Friction force was also modeled using spring and damper system.Marhefka and Orin[18] highlighted the limitation of linear contact model. They developed contact model using non-linear dissipation. Linear dissipation generates restoring forces not only in compression but also in tension. For evaluating dynamics of contact, an efficient and robust contact algorithm is required to detect contact region between the bodies in contact. The next section highlights the various contact algorithms developed in recent years.

III. CONTACT ALGORITHM AND RESPONSES

An efficient algorithm is needed to detect the contact surfaces and contact responses, and it should computationally fast while using insignificant memory. Munjiza and Andrews[19] highlighted requirements of an efficient contact algorithm as (1) minimization of computation time (2) minimization of RAM requirement. An algorithm can be body based searched or space based searched. They developed a no binary search (NBS) algorithm which is applicable to a large number of bodies moving in space, but the algorithm is limited to the bodies of same sizes. NBS algorithm is a space based searched algorithm. A two dimensional case was considered, the plane is divided into a number of square cells of size $2r$ each. The set of disc were mapped to the set of cells so that each disc is assigned one separate cell. Contacts were detected on the basis of the mapping. Any contact detection algorithm works in two stages: (1) neighbor search which is crude search to know possible bodies in contact. (2) Geometric resolution in which contacting bodies are examined in detail to know the point of contacts. For the bodies whose boundary can be expressed as implicit function f (x, y, z) = 0, a close form solution is available and geometric resolution algorithm is simple to develop, but it is cumbersome to develop contact algorithm for polygonal two dimensional or polyhedral three dimensional rigid bodies whose boundary cannot be expressed as an implicit function f (x, y, z) = 0. A fast common plane (CP) algorithm which converts the body-body contact problem to a body-plane contact problem was developed [20]. A common plane was considered in between the two contacting body. When two objects interact in the animation world, these can interpenetrate which is not desirable. If the animation reflects the real worlds, the contact should be detected and response of the contact must be modeled.Moore and Wilhelms[21] solved the kinematics problem of contact detection and the dynamics problem of the response of the contact. In their work, two algorithms to detect the collision were presented, and collision between the two arbitrary bodies was modeled using spring. Collision response algorithm was also presented that conserves linear and angular momentum. Two dimensional Cyrus-Beck Algorithm was considered that determines whether a point of one body lies within the geometry of another body. Dot product of unit vector normal to the edge $\hat{n}$ and vector $\overline{VP}$ is calculated as shown in Fig. 3. If it negative, it is

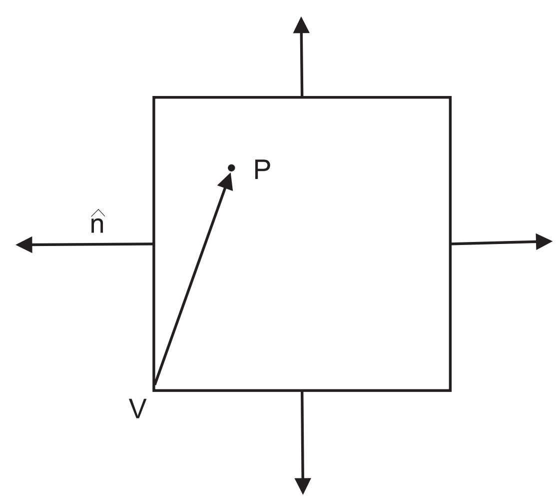

Fig. 3: Cyrus-Beck contact algorithm scheme.

inside the body. The vector from any corner and corresponding normal vector may be considered. Cyrus-Beck contact detection algorithm was extended in three dimensions which detects contact between two polyhedral rigid bodies.Choi*et al.*[22] developed general contact algorithm and compliant contact force model for contact response. In contact region, one body was allowed to penetrate the other.

Penetration depth and contact reference frame were determined from geometrical information of the rigid and flexible body surfaces. The contact algorithm was developed using main four steps: (1) surface representation (2) a pre-search (3) a detailed search (4) the contact force generation. The simple surface geometries may be represented by analytical function. In this study, a surface was represented by triangles or quadrilaterals. In pre-search, algorithm performed collision detection using special partitioning method. In detail search, depth of penetration and directions of contact reference frame were determined. The contact forces were calculated using compliant contact force model. Friction force was calculated using normal force and relative velocity between the two surfaces.Varadi*et al.*[23] developed a contact algorithm to study contact mechanics of rough surfaces. Using surface roughness data, the algorithm can determine the contact area and pressure distribution over it. The change in contact area during the sliding of the surfaces was also followed up. Zéhil and Gavin[24]developed an algorithm to solve steady state frictional rolling and sliding contact problem. The algorithm is applicable on a rigid cylinder or a rigid sphere that rolls on a layer of linear visco-elastic material of finite thickness. The algorithm determines the contact surface by resolving geometric conflicts and eliminating unacceptable surface tractions. Stick slip subroutine modeled the tangential frictional force. The model is not limited to finite thickness of the viscoelastic material, but can be adapted to handle deformable indenter. The iterative solving scheme can also be applied to the non-linear material.Baraff[25]calculated the forces between two polyhedral rigid bodies in static contact analytically. The analytic formulation used holonomic and non-holonomic constraints in consistent manner. The analytic solution used linear programming techniques to formulate and solved a system of equality and inequality constraints on the forces. As equations arise from penalty approach may be very stiff, it takes more computational time to solve it. The penalty approach gives approximate results as it allows interpenetration. Analytical method gives exact solution, but it is very difficult to implement. The limitation of the method is that linear programming software are complex as compared to the software used for solution of linear equations. Narwal *et al*[26]developed a general algorithm for contact detection and computed contact forces at the contact interface between a cylindrical disc and a layer of the soft material. Authors developed a bond graph model to evaluate the dynamics of soft contact interaction between a soft material and a rigid body. After detection of contact points, normal and tangential forces which act at the contact points are determined for dynamic simulation. Normal contact is taken viscoelastic and modeled using Kelvin-Voigt model. The dissipation in the model provides stable normal contacts. Most of the models discussed above do not take frictional force into account. The friction that acts at contact interface has been modeled in a number of ways; a literature review on the friction modeling is presented in the next section.

IV. FRICTION MODELING

For soft contact manipulation, frictional behavior at the contact interface is to be described. There is no exact formula available for the friction, and it is usually described by empirical models. A number of friction model which are found in the literature are studied. Three basic frictional models are: Coulombs, viscous and Stribeck friction models. Static frictional force cannot be expressed as a function of velocity, and even discontinuity at zero velocity makes it difficult to model. These models do not describe the presliding behavior in static limit of the friction.Karnopp[27] modeled the friction which explains stick-slip phenomenon. The stick-slip friction exists in most of the mechanisms and actuators. It is difficult to simulate stick-slip friction due to non-linear behavior in the vicinity of zero velocity. At zero velocity, the value of friction is more than its value at moderate velocity. Linear dynamic model is not valid for sticking phenomenon. A small region of velocity $-DV < V < DV$ was considered. In this region, Stick frictional force F_{Stick} was determined by other forces in the system. Outside this range of velocity, slip frictional force is some function of velocity F_{Stick} (V). From states equations, it was concluded that in stick region, the momentum was constant, and hence velocity was also constant with in $\pm DV$. Small value of velocity did not affect the results. The approach was applied to more complex systems and reasonable results were produced.Dahl[28] developed a model for rolling and sliding friction for simulations of physical systems dynamics. The model was developed on stress and strain analogy of the mechanics of material. He concluded that static friction is caused by contact bond stresses. For very small relative displacement at contact interface, elastic restoring force exits, and surfaces return to original position. He described spring like bond between two surfaces. For large displacement, the bonds rupture which result in permanent displacement. Contact bonds form and rupture as contact interface regularly. In rolling friction, friction involves tensile or compressive phenomenon while shearing is responsible for sliding friction. Within limit of static friction, presliding displacement is not apparent, and restoring force is proportional to small relative displacement. He concluded that frictional force is not only a function of velocity but also a function of relative displacement. Baraff[29] presented an algorithm which calculates contact forces with friction. Initially contact algorithm for frictionless surfaces was modeled analytically in which contact force is normal to the point of contact. Later friction was included, and the algorithm modeled static and dynamics frictions at different points at a time. Wit *et al.*[30] described model based friction comprehensive techniques. They observed that in precision positioning and low velocity tracking, the model based upon Coulombs or viscous friction does not give exact result. They proposed a frictional model which includes Stribeck effect in the Dahl's friction model. The asperities on the contact surfaces were represented as elastic bristles. These bristles deflect like spring and give rise to frictional force.**S**tick-slip phenomenon of finger pad on a smooth wet glass was investigated [31]. Normal and tangential forces along with coefficient of static friction were determined. It was found that stationery as well as stick-slips friction decrease systematically as a function of normal load and sliding velocity. Overall model for the friction agrees with the adhesive model of the friction. The results presented on stick slip friction in the paper may be useful for development of skin for robotic hands, and may improve the control of the grip task and dexterous manipulation of the object by robotic hands.Tomlinson*et al.*[32] investigated the relation between normal force and frictional force for a human finger. The relation was examined with the help of 12 materials: 3 metals, 5 plastics and 4 elastomers. Each material had been subjected to wide range of normal forces in 40 tests. The effect of surface roughness on the contact friction was also examined, and tests were conducted on brass, steel and aluminum with wide range of surface roughness. The deformation of the finger was monitored using high speed camera. It was concluded that for a nominally smooth surface, the relation between normal force and frictional force is linear above 1 *N* normal force. At low roughness, there is very small difference between the coefficient of friction for different material, but as the surface roughness increases, the co-efficient of friction increases up to a certain point. Rajaei and Ahmadian[33]developed a model to see the response of a beam with frictional support. In contact mechanics, friction is generally modeled considering normal force to be constant. But in dynamic contact, relative normal motion in addition to tangential motion also exists which changes the normal force. This phenomenon was observed as high amplitude lateral vibrations exist during dynamic contact. Avlonitis*et al.*[34]modified the Olami-Feder-Christensen (OFC) friction spring block model to investigate the sliding surfaces which involve partial contact. Each model asperity contact was assumed to experience a static force threshold. In static friction zone, there was no macro sliding between the two surfaces, but there was micro relative displacement. The model can estimate the real contact area between two sliding surfaces. Deladi[35]modeled static friction between a rough rubber and a metal surface. The rubber was assumed to be viscoelastic in nature. The model considered single viscoelastic-rigid asperity coupling. The asperities stick at the center and slides at the annulus of the contact area. As the tangential force increases, the slip area increases. The limit of static friction is up to the limit when slip area equalizes the total contact area. The single asperity model was then extended to multiple asperities. The two surfaces which were under consideration were subjected to normal and tangential forces.Narwal *et al.*[36]modeled friction using Kelvin-Voigt model. A spring and a damper in Kelvin-Voigt configuration are inserted between the contact node S_i on the soft material and the contact point P_i on the rigid body. Within limit of static friction, the point P_i remains attached with the node S_i with spring and damper in between. As the tangential force exceeds the limit of static friction, the viscoelastic bond between the point P_i and the node S_i breaks. Two surfaces slide and dynamic friction acts at the contact interface. Bond graph model which is developed by authors are simulated with different geometries[37]. The model determines contact area and distribution of contact forces for both types of contacts when contact interface is stationery and when it moves in case of rolling or sliding of rigid body over the soft material. In the light of the literature survey, development in the field of contact mechanics is concluded in the next section with its proposed future extension.

V. CONCLUSION

Most of the models are developed for simple geometry, and static contacts when two bodies are at rest are considered. Evaluation of dynamics of contact becomes quite challenging when there is rolling or sliding contact. Modeling of friction at the moving contact interface in case of rolling or sliding is complex. Fast and robust contact algorithm is required to detect contacts and its response. Bond graph is graphical representation of physical system dynamics. Causality based representation of bonds elucidates the soft contact interaction. Bond graph model for soft contact is developed for two dimensions which is applicable to all geometries. The model is also applicable to colliding contacts. The approach elucidates soft contact interaction and seems very useful in the area of contact mechanics, and can be extended in three dimensions. The work carried by authors is proposed to be extended for soft contact manipulation and development of prosthetic fingers for rehabilitation.

REFERENCES

[1]. Hertz Heinrich, “On the Contact of Elastic Solid,” *J. reine angew . math*, vol. 92, pp. 156–171, 1881.

[2]. K. L. Johnson, K.Kendall, and A. D. Roberts, “Surface energy and the contact of elastic solids,” in *Royal society of london*, 1971, vol. 324, no. 1558, pp. 301–313.

[3]. B. . Derjaguin, V. . Muller, and Y. . Toporov, “Effect of contact deformations on the adhesion of particles,” *Journal of Colloid and Interface Science*, vol. 53, no. 2, pp. 314–326, Nov. 1975.

[4]. N. Xydas and I. Kao, “Modeling Of Contact Mechanics With Experimental Results For Soft Fingers,” in *IEEE International Conference on Intelligent Robots and System*, 1998, no. October, pp. 488–493.

[5]. S. H. Bakhy, S. S. Hassan, S. M. Nacy, K. Dermitzakis, and A. H. Arieta, “Contact mechanics for soft robotic fingers: modeling and experimentation,” *Robotica*, vol. 31, no. 04, pp. 599–609, Oct. 2012.

[6]. M. Machado, P. Moreira, P. Flores, and H. M. Lankarani, “Compliant contact force models in multibody dynamics: Evolution of the Hertz contact theory,” *Mechanism and Machine Theory*, vol. 53, pp. 99–121, Jul. 2012.

[7]. K. Namima, Z. Wang, and S. Hirai, “Simulation of soft fingertip deformation under contact and rolling constraints using FEM and CSM,” *2009 IEEE International Conference on Robotics and Biomimetics (ROBIO)*, pp. 1585–1590, Dec. 2009.

[8]. J. P. Gourret, D. Thalmann, and N. Magnenat-Thalmann, “Modeling of contact deformations between a synthetic human and its environment,” *CAD*, vol. 23, no. September, 1991, pp. 514–520, 1991.

[9]. A. Vaz and A. K. Maini, “Modeling of Soft Materials/ : Integrating Bond Graphs with Finite Element Analysis,” in *14th National Conference on Machines and Mechanism*, 2009, no. 1, pp. 247–252.

[10]. N. Xydas, M. Bhagavat, and I. Kao, “Study of Soft-Finger Contact Mechanics Using Finite Elements Analysis and Experiments,” in *IEEE International Conference on Robotics and Automation*, 2000, no. April, pp. 2179–2184.

[11]. Y. Alinia, M. a. Guler, and S. Adibnazari, “On the contact mechanics of a rolling cylinder on a graded coating. Part 1: Analytical formulation,” *Mechanics of Materials*, vol. 68, pp. 207–216, Jan. 2014.

[12]. K. Yamane and Y. Nakamura, “Stable penalty-based model of frictional contacts,” in *IEEE International Conference on Robotics and Automation*, 2006, no. May, pp. 1904–1909.

[13]. S. Kucharski and G. Starzynski, “Study of contact of rough surfaces: Modeling and experiment,” *Wear*, vol. 311, no. 1–2, pp. 167–179, Mar. 2014.

[14]. S. Moisio, B. León, P. Korkealaakso, and A. Morales, “Model of tactile sensors using soft contacts and its application in robot grasping simulation,” *Robotics and Autonomous Systems*, vol. 61, no. 1, pp. 1–12, 2013.

[15]. M. Pau, F. Aymerich, and F. Ginesu, “Distribution of contact pressure in wheel–rail contact area,” *Wear*, vol. 253, no. 1–2, pp. 265–274, Jul. 2002.

[16]. S. Hirai, K.-H. Park, and B.-H. Kim, “Development of a soft-fingertip and its modeling based on force distribution,” in *2003 IEEE International Conference on Robotics and Automation*, 2003, pp. 3169–3174.

[17]. Y. Hwang, E. Inohira, A. Konno, and M. Uchiyama, “An Order n Dynamic Simulator for a Humanoid Robot with a Virtual Spring-Damper Contact Model,” in *IEEE International Conference on Robotics and Automation*, 2003, pp. 31–36.

[18]. D. W. Marhefka and D. E. Orin, “Simulation of Contact Using a Nonlinear Damping Model,” in *IEEE International Conference on Robotics and Automation, Minnesota*, 1996, no. April, pp. 1662–1668.

[19]. A. Munjiza and K. R. F. Andrews, “NBS contactdetection algorithm for bodies of similar size,” vol. 149, no. February, pp. 131–149, 1998.

[20]. E. G. Nezami, Y. M. a. Hashash, D. Zhao, and J. Ghaboussi, “A fast contact detection algorithm for 3-D discrete element method,” *Computers and Geotechnics*, vol. 31, no. 7, pp. 575–587, Oct. 2004.

[21]. M. Moore and J. Wilhelms, “Collision Detection and Response for Computer Animation,” *Computer Graphics*, vol. 22, no. 4, pp. 289–298, 1988.

[22]. J. Choi, S. Rhim, and J. H. Choi, “A general purpose contact algorithm using a compliance contact force model for rigid and flexible bodies of complex geometry,” *International Journal of Non-Linear Mechanics*, vol. 53, pp. 13–23, Jul. 2013.

[23]. K. Varadi, K. Friedrich, and Z. Neder, “Evaluation of the real contact areas , pressure distributions and contact temperatures during sliding contact between real metal surfaces,” *wear*, vol. 200, pp. 55–62, 1996.

[24]. G.-P. Zéhil and H. P. Gavin, “Simple algorithms for solving steady-state frictional rolling contact problems in two and

three dimensions," *International Journal of Solids and Structures*, vol. 50, no. 6, pp. 843–852, Mar. 2013.

[25]. D. Baraff, "Analytical methods for dynamic simulation of non-penetrating rigid bodies," *Computer Graphics*, vol. 23, no. 3, pp. 223–232, Jul. 1989.

[26]. A. K. Narwal, A. Vaz, and K. D. Gupta, "Study of dynamics of soft contact rolling using multibond graph approach," *Mechanism and Machine Theory*, vol. 75, pp. 79–96, May 2014.

[27]. D. Karnopp, "Computer Simulation of Stick-Slip Friction in Mechanical Dynamic System," *Journal of Dynamic System, Measurement and Control, Transection of ASME*, vol. 107, no. March, pp. 100–103, 1985.

[28]. P. R. Dahl, "Solid Friction Model," 1968.

[29]. D. Baraff, "Fast Contact Force Computation for Nonpenetrating Rigid Bodies," in *Computer Graphics*, 1994, no. May, pp. 23–34.

[30]. C. C. De Wit, H. Olsson, k. J. Astrom, and P. Lischinsky, "A New Model for Control of Systems with Friction," *IEEE Transaction on Automatic Control*, vol. 40, no. 3, pp. 419–425, 1995.

[31]. S. Derler and G.-M. Rotaru, "Stick–slip phenomena in the friction of human skin," *Wear*, vol. 301, pp. 324–329, Apr. 2013.

[32]. S. E. Tomlinson, R. Lewis, and M. J. Carré, "The effect of normal force and roughness on friction in human finger contact," *wear*, vol. 267, pp. 1311–1318, 2009.

[33]. M. Rajaei and H. Ahmadian, "Development of generalized Iwan model to simulate frictional contacts with variable normal loads," *Applied Mathematical Modelling*, pp. 1–13, 2014.

[34]. M. Avlonitis, K. Kalaitzidou, and J. Streator, "Investigation of friction statistics and real contact area by means of a modified OFC model," *Tribiology International*, vol. 69, pp. 168–175, 2014.

[35]. E. L. Deladi, M. B. de Rooij, and D. J. Schipper, "Modelling of static friction in rubber–metal contact," *Tribology International*, vol. 40, no. 4, pp. 588–594, Apr. 2007.

[36]. A. K. Narwal, A. Vaz, and K. D. Gupta, "Evaluation of Dynamics of Sfot Contact Rolling using Multibond Graph Approach," in *1st international and 16th national conference on machines and mechanisms*, 2013, pp. 23–29.

[37]. A. K. Narwal, K. D. Gupta, and A. Vaz, "Understanding Soft Contact Interaction between a Non Circular Rigid Body and a Soft Material Using Multibond Graph," in *International Conference on Bond Graph Modeling*, 2014, pp. 690–697.

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